Bronx Community College of City University of New York
Department of Mathematics and Computer Science

MATH 13 REVIEW SHEET

1. Find the x and y components A_x and A_y of the given vectors. Round to an appropriate decimal place according to the number given in each problem.
(a) $A = 1.884, \theta = 58.24°$ (b) $A = 24.15, \theta = 350°$ (c) $A = 55.0, \theta = 122°$

2. Add the vectors $9\angle 35°, 45\angle 256°$, and $56\angle 300°$, and find the magnitude and the direction of the resultant vector to the nearest tenth.

3. A space shuttle is moving in orbit at 18,250 mph. A satellite is launched to the rear at 150 mph at an angle of 6.75° from the direction of the shuttle. Find the velocity of the satellite.

4. A river flows at the rate of 5 km/h, A rower who can travel 7.6 km/h in stillwater, heads directly across the current. Find the rate and direction of travel of the boat to the nearest tenth.

5. To avoid a storm, a plane flies 125 mi south, then 85 mi at 35.0° south of west, and finally 240 mi at 48.0° north of west. What is the displacement of the plane from its original position?

6. Perform the indicated operations. Express all answers in the form $a + bj$.
(a) $5 + \sqrt{-\frac{16}{25}} - \sqrt{-25} + 3$ (b) $(34 + 8j) - (6 - j)$ (c) $(6 + j)(4 - 2j)$ (d) $\dfrac{5 - 2j}{1 - 2j}$
(e) $j^{17} - 7j^8$

7. For each given complex number fill in the blank. Give answers in exact values whenever possible.

rectangular form	polar form	exponential form
$2 - \sqrt{5}j$		
		$6e^{\frac{7\pi j}{4}}$
	$2\angle 150°$	

8. Perform the follwing indicated operations and give the answers in appropriate form:
(a) $(6e^{2.3j})(2e^{-2.3j})$ (b) $\dfrac{20(\cos 35° + j \sin 35°)}{4(\cos 55° + j \sin 55°)}$ (c) $(2\angle 54°)^6$ (d) $(\frac{1}{2} - \frac{\sqrt{3}}{2}j)^{10}$

9. (a) Find all three cube roots of $-27j$ and express your answers in standard polar form.
(b) Find all four 4th roots of $-\frac{1}{2} + \frac{\sqrt{3}}{2}j$ and express your answers in rectangular form.

10. Find the real values of x and y satisfying the equation:

(a) $5-(2y)j = 6j-(x-1)$ (b) $(x^2-2x)+yj = 3+(y^2)j$

11. Let $f(x) = \dfrac{9-x^2}{2x}$, find $f(\sqrt{5}), f(-2), f(0)$, and $f(x-3)$.

12. Determine the domain of each function.

(a) $y = \dfrac{x}{x^2-9}$ (b) $y = \sqrt{x-3}-2$ (c) $y = 2x^2-4x+6$ (d) $y = 4^x$ (e) $y = \log_3 x$

13. Sketch the graph of each equation.

(a) $y = -x^2+2x-3$ (b) $y = \sqrt{4-x}$

14. Sketch the graph of each of the following function.

(a) $f(x) = 2^x$ (b) $f(x) = (\frac{1}{2})^x$ (c) $g(x) = \log_2 x$

15. Fill in the blank.

exponential form	$2^{-3} = \frac{1}{8}$			$x^y = z$
logarithmic form		$\ln x = 2.1$	$\log 0.001 = -3$	

16. Write each logarithm as the sum or difference of two or more terms:

(a) $\log_5 135$ (b) $\log\left(\dfrac{5a^3x}{3-y}\right)$ (c) $\ln\sqrt{\dfrac{xy+y}{x^2-1}}$

17. Simplify each expression to a single logarithm:

(a) $3\ln 5 - 5\ln 3$ (b) $\log 4 + \dfrac{\log x}{2} - 2\log y - \log\dfrac{3}{4}$

18. Solve for x

(a) $2^x = 3^{x-1}$ (b) $\log 2x + \log 5x = 1$ (c) $3\log(2x-1) = 2$ (d) $2e^{2x-1} = e^{5x}$

19. Write the amplitude, period, and phase shift of y. Draw one cycle of the graph and label the coordinates of the maximum, minimum and intercept points.

(a) $y = -3\cos(2\pi x)$ (b) $y = 2\sin(2x+\pi)$ (c) $y = -2\cos(\dfrac{x}{2}-\dfrac{\pi}{2})$ (d) $y = -2\tan(4x-3)$

20. Prove each identity:

(a) $(\tan x + \cot x)\sin x \cos x = 1$

(b) $\sec x + \tan x + \cot x = \dfrac{1 + \sin x}{\cos x \sin x}$

(c) $\dfrac{\sin(x - y)}{\sin(x + y)} = \dfrac{\tan x - \tan y}{\tan x + \tan y}$

(d) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

21. Solve the given equations for all values of $0 \leq x < 2\pi$.

(a) $2\cos^2 x = \cos x$ (b) $2\cos^2 x - \sin x - 1 = 0$ (c) $(\tan x - 1)(\tan x - \sqrt{3}) = 0$
(d) $\sin 2x \sin x = \cos x$

22. Find the value of each determinant. Simplify your answers whenever possible.

(a) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ (b) $\begin{vmatrix} \sqrt{2} & 2 \\ \sqrt{6} & \sqrt{3} \end{vmatrix}$ (c) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 8 & 9 & 4 \end{vmatrix}$ (d) $\begin{vmatrix} \tan\theta & -\tan\theta \\ \tan\theta & \cot\theta \end{vmatrix}$

(e) $\begin{vmatrix} e^0 & e^{\sin^2 x} \\ e^{\cos^2 x} & e \end{vmatrix}$

23. Use Cramer's rule to solve each linear system.

(a) $\begin{cases} 4x + y + 3z = -5 \\ 2x - 2y + 6z = 6 \\ x + 3y + 3z = 11 \end{cases}$ (b) $\begin{cases} a + b - 3c = 7 \\ -2a + b + 2c = -1 \\ a + 4b + c = 12 \end{cases}$

Answers

1. (a) $A_x = 0.992, A_y = 1.602$ (b) $A_x = 23.78, A_y = -4.19$ (c) $A_x = -29.1, A_y = 46.6$

2. $A = 90.4, \theta = 285.7°$

3. 18,101.04 mph and 0.0558° from the direction of the shuttle.

4. $v = 9.1$km/h, and $\theta = 33.3°$ away from the original direction of the boat

5. 230 miles, 178.9° or 1.1° north of west.

6. (a) $8 - \frac{21}{5}j$ (b) $28 + 9j$ (c) $26 - 8j$ (d) $\frac{9}{5} + \frac{8}{5}j$ (e) $-7 + j$

7.

rectangular form	polar form	exponential form
$2 - \sqrt{5}j$	$3\angle 312°$	$3e^{5.44j}$
$3\sqrt{2} - 3\sqrt{2}j$	$6\angle 315°$	$6e^{\frac{7\pi}{4}j}$
$-\sqrt{3} + j$	$2\angle 150°$	$2e^{\frac{5\pi}{6}j}$

8. (a) 12 (b) $5(\cos 340° + j \sin 340°)$ (c) $64\angle 324°$ (d) $-\frac{1}{2} + \frac{\sqrt{3}}{2}$

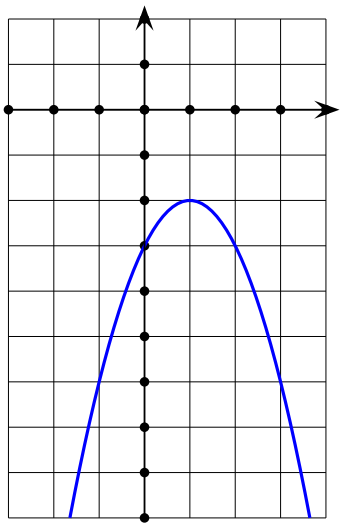
9. (a) $3\angle 90°, 3\angle 210°, 3\angle 330°$ (b) $\frac{\sqrt{3}}{2} + \frac{1}{2}j, -\frac{1}{2} + \frac{\sqrt{3}}{2}j, -\frac{\sqrt{3}}{2} - \frac{1}{2}j, \frac{1}{2} - \frac{\sqrt{3}}{2}j$

10. (a) $x = -4; y = -3$ (b) $x = 3, -1; y = 0, 1$

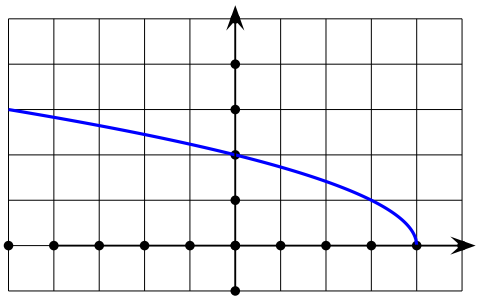
11. $f(\sqrt{5}) = \frac{2\sqrt{5}}{5}; f(-2) = -\frac{5}{4}; f(0)$ is undefined; $f(x-3) = \frac{-x^2+6x}{2x-6}$

12. (a) all real numbers except $-3, +3$ or $(-\infty, -3) \cup (-3, +3) \cup (+3, +\infty)$
    (b) $[3, +\infty)$ (c) $(-\infty, +\infty)$ (d) $(-\infty, +\infty)$ (e) $(0, +\infty)$

13.

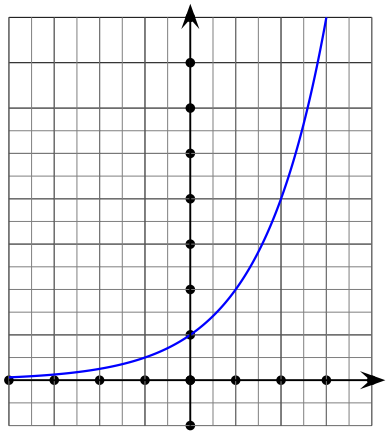


(a)

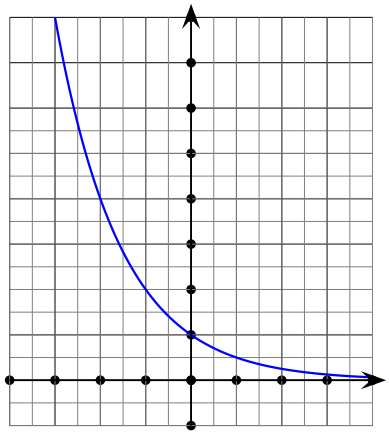


(b)

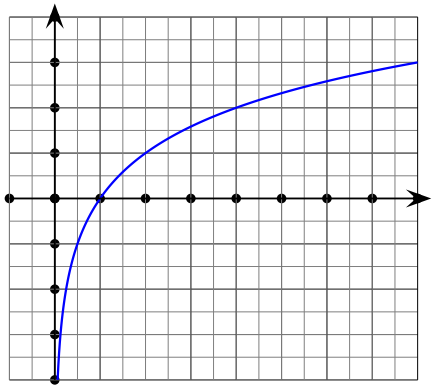
14.



(a)



(b)



(c)

15.

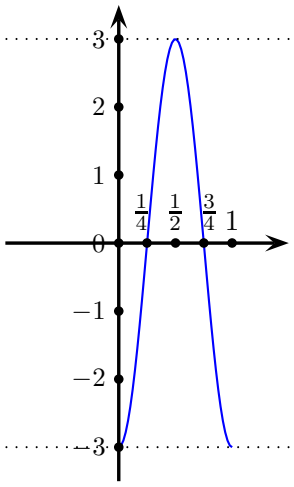
exponential form	$2^{-3} = \frac{1}{8}$	$e^{2.1} = x$	$10^{-3} = 0.001$	$x^y = z$
logarithmic form	$\log_2(\frac{1}{8}) = -3$	$\ln x = 2.1$	$\log 0.001 = -3$	$\log_x z = y$

16. (a) $1 + 3\log_5 3$ (b) $\log 5 + 3\log a + \log x - \log(3 - y)$ (c) $\frac{1}{2}\ln y - \frac{1}{2}\ln(x - 1)$

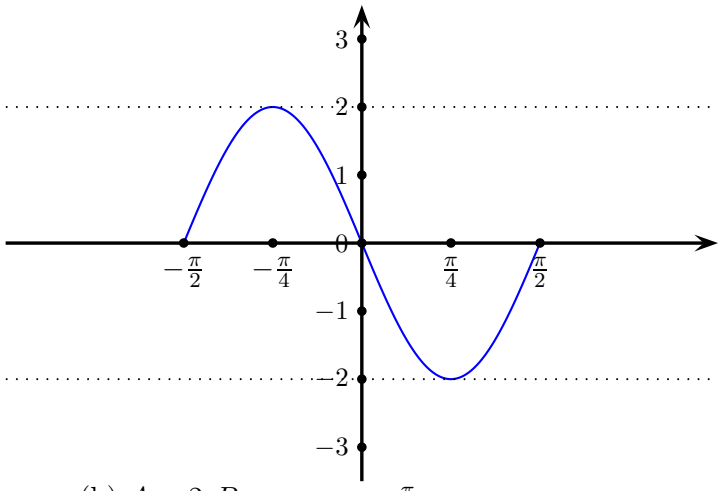
17. (a) $\ln\left(\frac{125}{243}\right)$ (b) $\log\left(\frac{16\sqrt{x}}{3y^2}\right)$

18. (a) $\frac{-\ln 3}{\ln(\frac{2}{3})}$ or $\frac{\ln 3}{\ln(\frac{3}{2})}$ (b) 1 (c) $\frac{1+\sqrt[3]{100}}{2}$ (d) $\frac{\ln 2 - 1}{3}$

19.

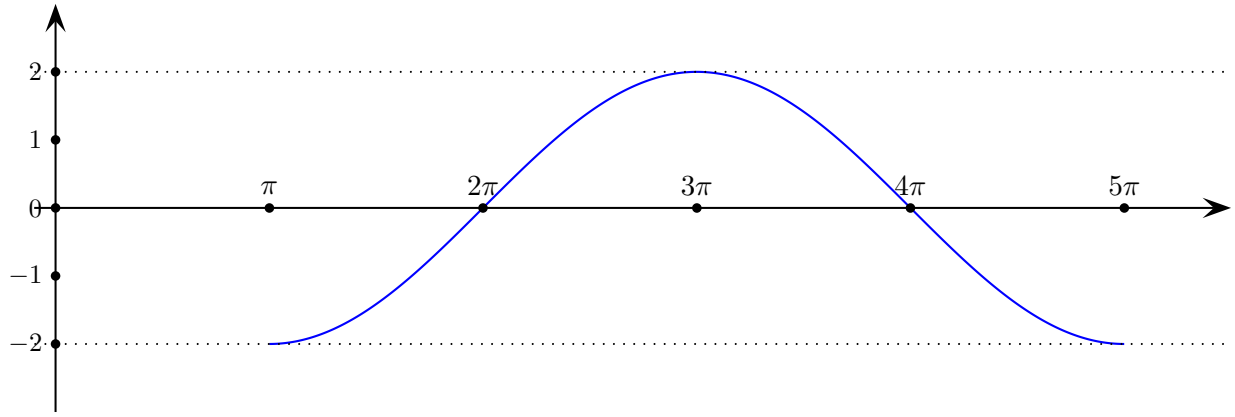


(a) $A = 3; P = 1; ps = 0$

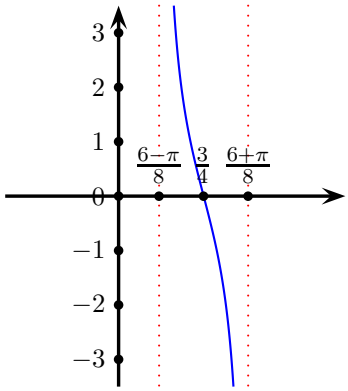


(b) $A = 2; P = \pi; ps = -\frac{\pi}{2}$

(c) $A = 2; P = 4\pi; ps = \pi$



(d) No amplitude, $P = \frac{\pi}{4}; ps = \frac{3}{4}$



21. (a) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ (b) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ (c) $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{\pi}{3}, \frac{4\pi}{3}$ (d) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

22. (a) 1 (b) $-\sqrt{6}$ (c) 0 (d) $\sec^2\theta$ (e) 0

23. (a) $(-4, 2, 3)$ (b) $(1, 3, -1)$

N.A. 2010, P.Y. 2012