WORKBOOK. MTH 13 -TRIGNOMETRY AND COLLEGE ALGEBRA (3 CREDITS, 4 HOURS)

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

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Contents

1. Introduction to Functions

(1) Define a function. Give five examples; use different variables for greater understanding. (Note to instructor: Until otherwise mentioned, a function in this course is real-valued, and defined on a subset of $\mathbb{R}$.)

(2) Present at least two different examples of relations which are not functions.

(3) What are the independent and dependent variables in the description of a function? Point out the independent and dependent variables in your examples above.

(4) Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = \dfrac{2x}{x^2+1}$. Evaluate

• $f(0)$ • $f(1)$ • $f(-1)$ • $f(2)$ • $f(-2)$.

(5) Let $f : \mathbb{R} \to \mathbb{R}$ be the function given by $f(x) = \dfrac{1}{x^2+2}$. Find

• $f(a)$ • $f(b)$ • $f(-1)$ • $f(a^2)$ • $f(\sqrt{b})$.

(6) Let $g(a) = \sqrt{a^2 - 3}$. Find

- $g(10)$ • $g(-10)$ • $g(2)$ • $g(b^2)$ • $g(\sqrt{5})$.

(7) Let $h(m) = 3 - 4m + 5|m|$. Find

- $h(10)$ • $h(-10)$ • $h(2)$ • $h(0)$ • $h\left(-\frac{1}{5}\right)$.

(8) Let $f(x) = 2x - 5$. Find

- $f(a+2)$ • $f(a+h)$ • $f(a+h) - f(a)$ • $f(2s) - 2f(s)$ • $f\left(\frac{1}{x}\right)$.

(9) Let $f(x) = x^{12} - 3$. Find $f(2.345)$. (Use a calculator.).

(10) The length of a rectangle is twice its width. Describe the area of the rectangle as a function of its width.

(11) Describe the volume of a sphere as a function of its radius.

(12) Describe the radius of a sphere as a function of its volume.

(13) Express the area of an equilateral triangle as a function of its side length s.

2. More about Functions

(1) Write down five examples of functions.

(2) Define the domain of a function. What is the domain of each of the functions given above?

(3) Define the range of a function. What is the range of each of the functions given above?

(4) Find the domain of the following functions:

- $f(x) = 3x + 5$

- $g(v) = 4v^2 - 7$

- $h(m) = \sqrt{m}$

- $f(t) = \dfrac{3}{t} + 5$

- $g(w) = \dfrac{w+3}{w-3}$

- $h(x) = \dfrac{3}{x} - \dfrac{x+4}{x-4} + \sqrt{x}$

- $k(x) = \sqrt{5-x}$

- $f(x) = \dfrac{x+2}{\sqrt{x-3}}$

- $g(t) = \dfrac{3}{t+5} - \dfrac{4}{t-1}$

- $h(s) = \dfrac{\sqrt{s+2}}{s-3}$

(5) Find the range of the following functions:

- $f(x) = 3x + 5$
- $g(m) = \sqrt{m}$
- $h(s) = \sqrt{3s + 5}$
- $k(t) = 3t^4 + t^2 + 1$
- $f(x) = \dfrac{3}{x}$
- $g(v) = |v + 3|$
- $h(n) = \sqrt{5 - n}$
- $t(x) = x + |x|$

(6) Evaluate $f(1)$ and $f(2)$ where

- $f(x) = \begin{cases} 3x + 5x^2 & \textit{for } x \leq 1 \\ 4x - 2 & \textit{for } x > 1 \end{cases}$

- $f(t) = \begin{cases} \sqrt{3t + 5} & \textit{for } t > 1 \\ \dfrac{2}{t - 2} & \textit{for } t \leq 1 \end{cases}$

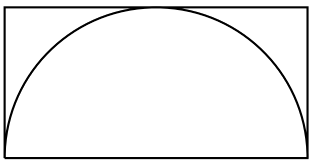
- $f(s) = \begin{cases} \sqrt{s + 2} & \textit{for } s > 1 \\ s & \textit{for } s \leq 1 \end{cases}$

- $f(m) = \begin{cases} \dfrac{m + 2}{m - 1} & \textit{for } m \neq 1 \\ 3 & \textit{for } m = 1 \end{cases}$

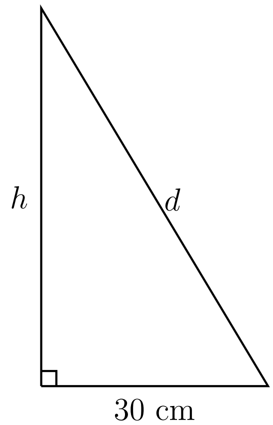
(7) A right circular cylinder has height 15 m.

- Describe the volume of the cylinder as a function $f(r)$ of its radius r.
- Find the volume when the radius is 10 m.
- What should the radius be for the volume to be 4500π m^3?

(8) The length of a rectangle is twice its width. A semicircle is drawn within the rectangle with the longer side as its diameter. Describe the area of the region between the rectangle and the semicircle as a function of the width.



(9) Write the height h as a function of the distance d in the adjoining figure.



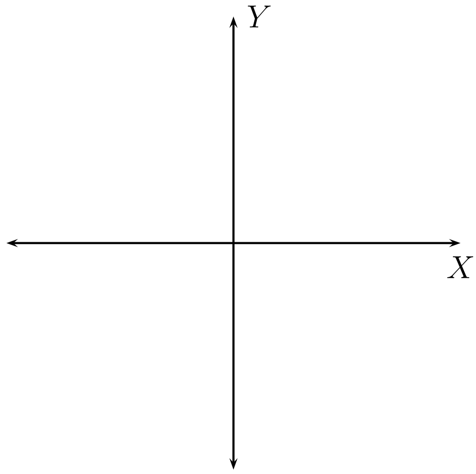
(10) Tom has opened a savings bank account. For the first two years he intends to deposit \$ 300 per month in this account. After that he intends to deposit \$ 400 per month. Write the total savings (s) in terms of the number of months (m) when $m \geq 24$, where $m = 1$ represents the first month he starts saving.

(11) One day Mr. Banker finds one penny on the street and puts it in a box. Next day he again finds a penny and puts it in the box. Since then everyday he finds as many pennies on the street as he already has in the box, and everyday he adds these pennies to the box. Describe the total amount of money (A) in the box as a function of the number of days (d) where $d = 1$ represents the first day he started collecting these pennies.

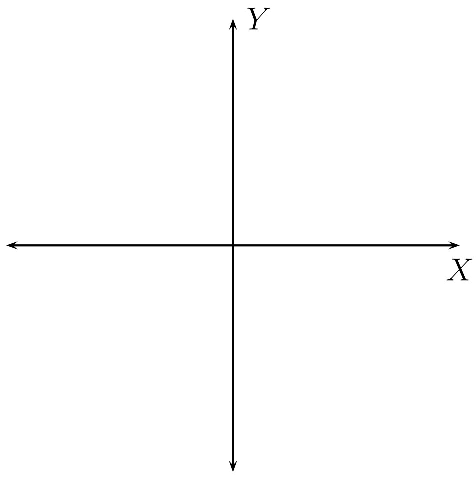
3. Graph of a function

Graph the following functions. Discuss the vertical line test, domain, range, and plot at least four points for each of the graph.

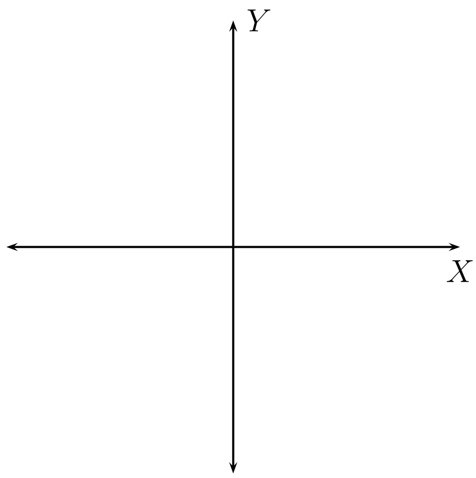
(1) $y = 1$ (constant function)



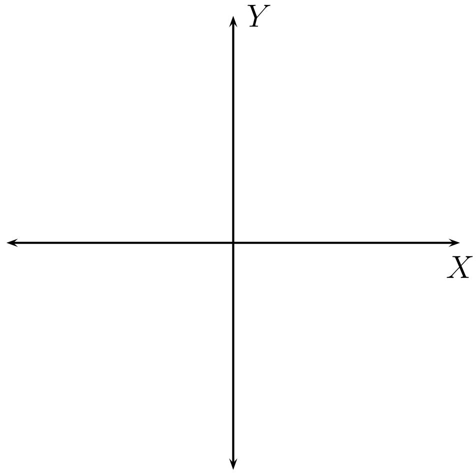
(2) $y = x$ (the basic linear function)



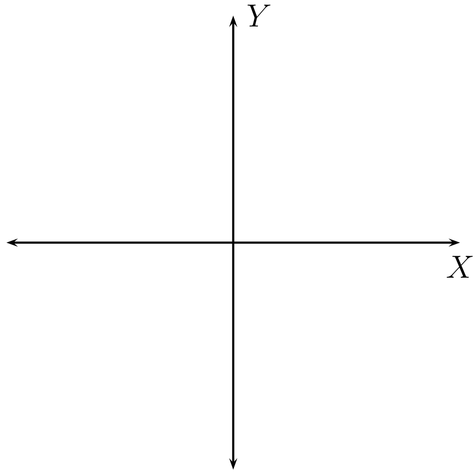
(3) $y = x^2$ (the basic parabola)



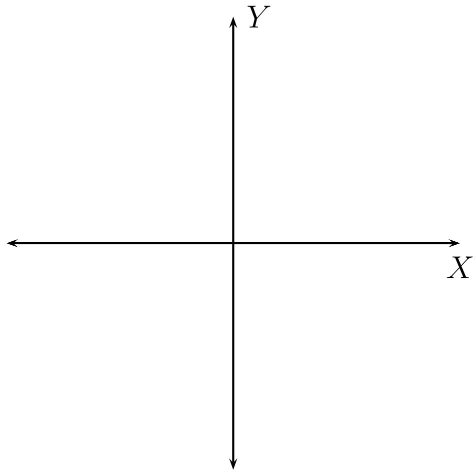
(4) $y = x^3$ (the basic cubic)



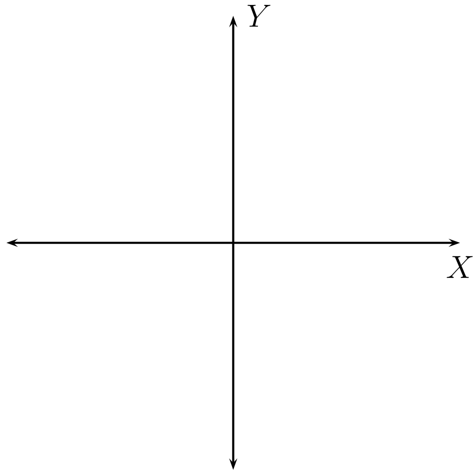
(5) $y = \sqrt{x}$ (the basic square-root)



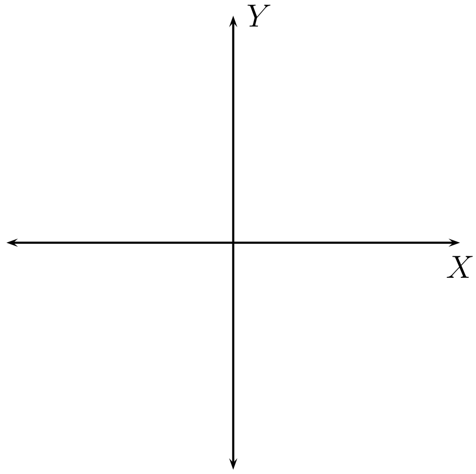
(6) $y = \sqrt[3]{x}$ (the basic cube-root)



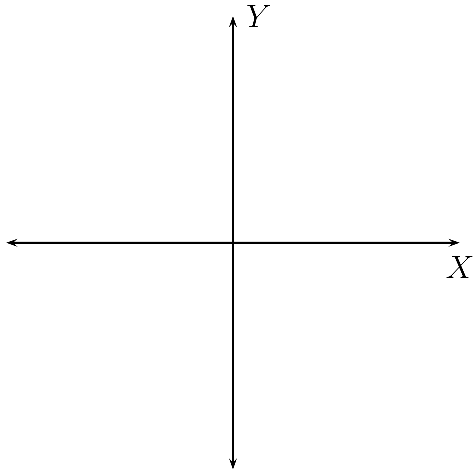
(7) $y = |x|$ (the absolute-value function)



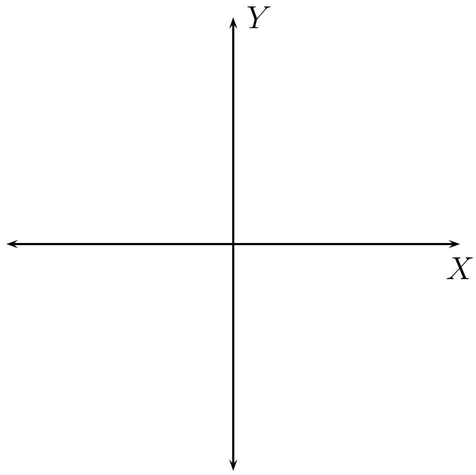
(8) $y = \dfrac{1}{x}$ (the basic hyerbola)



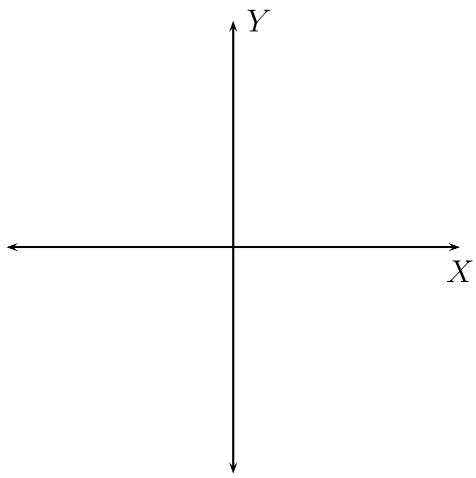
(9) $y = \dfrac{1}{x^2}$



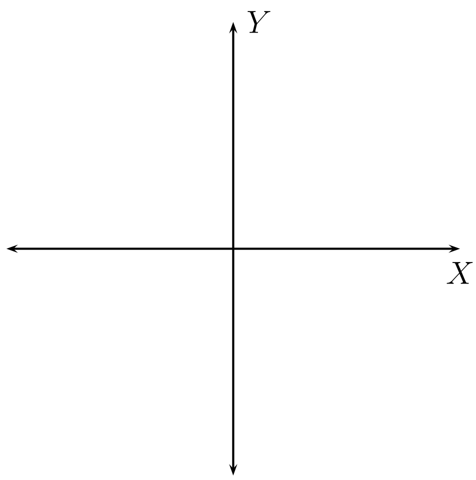
(10) $y = 3x + 5$



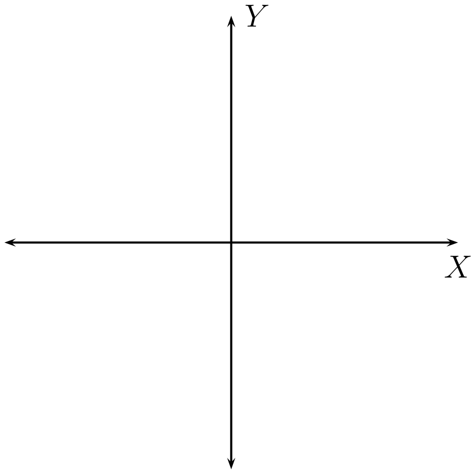
(11) $y = -\dfrac{1}{2}x + 5$



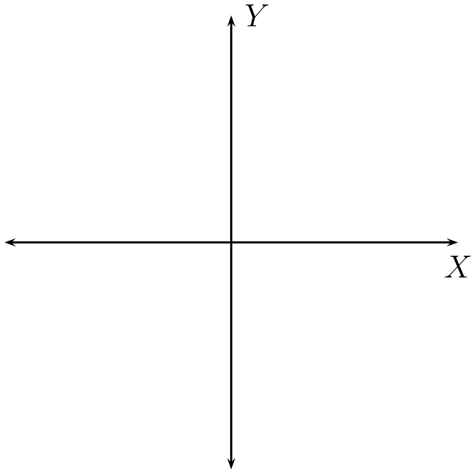
(12) $y = 3x^2 - 2$



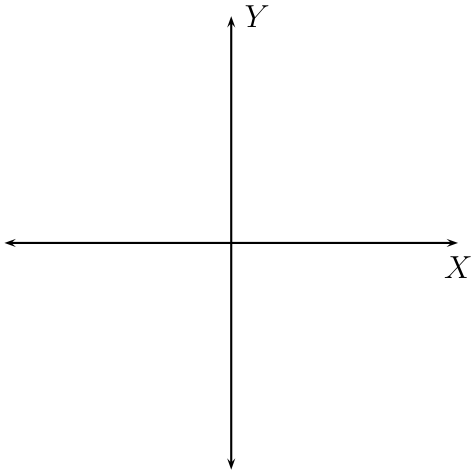
(13) $y = \sqrt{x+5}$



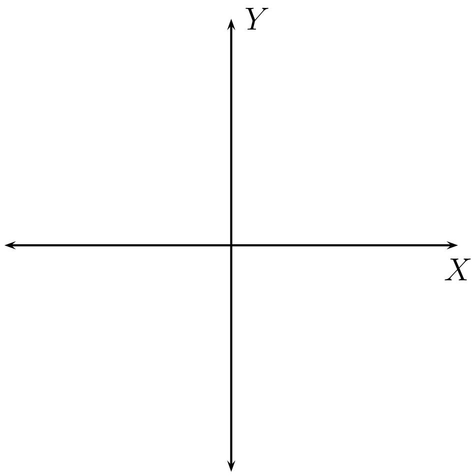
(14) $y = \sqrt{x-5}$



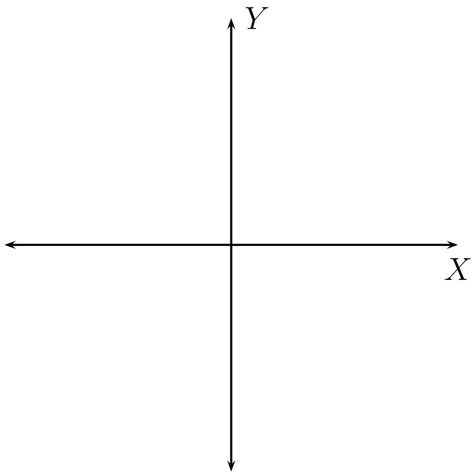
(15) $y = \sqrt{4-x}$



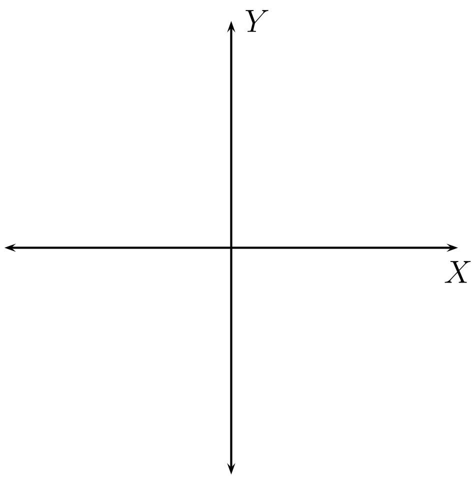
(16) $y = x^2 + 2x + 1$



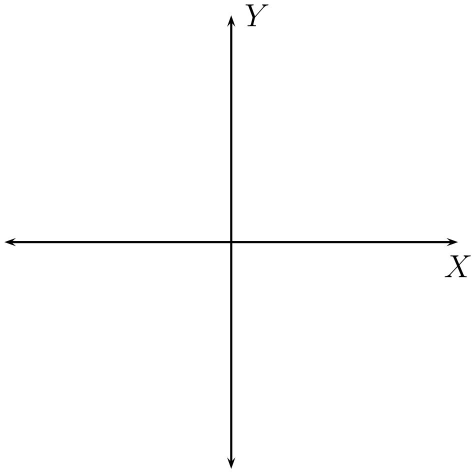
(17) $y = -2x^2 + 4x + 3$



(18) The piece-wise defined function $f(x) = \begin{cases} x & x \leq 1 \\ x^2 & x > 1 \end{cases}$



(19) The piece-wise defined function $g(x) = \begin{cases} 3x & x < -1 \\ 3 & -1 \leq x \leq 2 \\ -x^2 + 3 & x > 2 \end{cases}$



4. Exponential Functions

The Exponential function:

$$f(x) = b^x \text{ for } b > 0 \text{ and } b \neq 1.$$

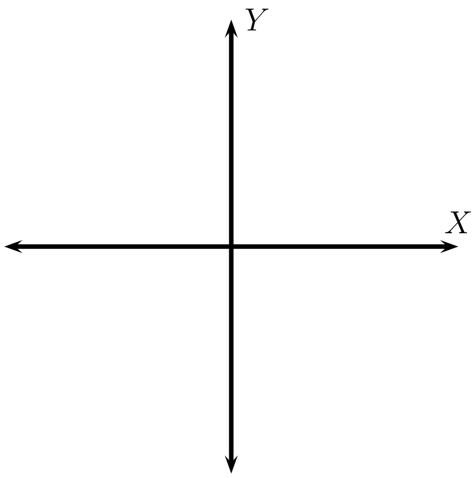
Here, b is the **base** of the exponential function.

What happens when $b = 1$? What is its graph?

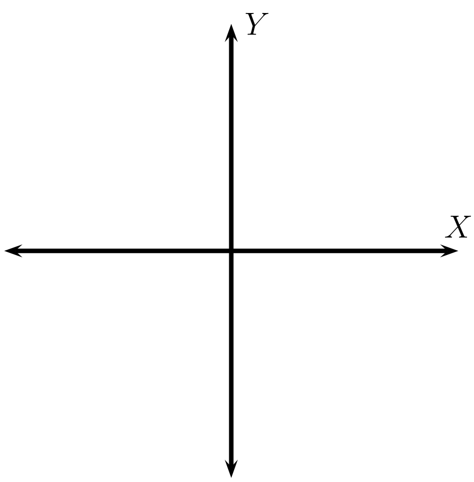
What happens when $b < 0$? What about $b = 0$?

Graph the following exponential functions. Provide at least 5 points. What are its x or x intercepts if any. Give its domain and range. Sketch the horizontal asymptote as a dotted line, and give its equation.

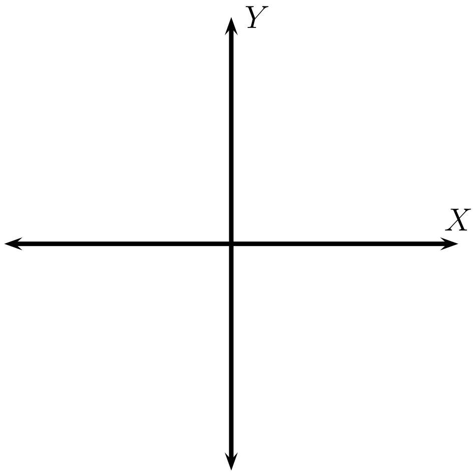
(1) $f(x) = 2^x$



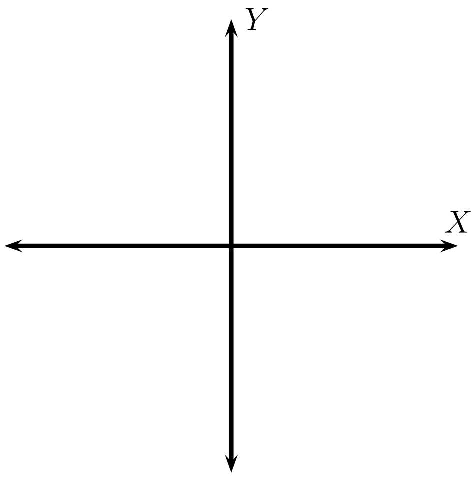
(2) $f(x) = 3^x$



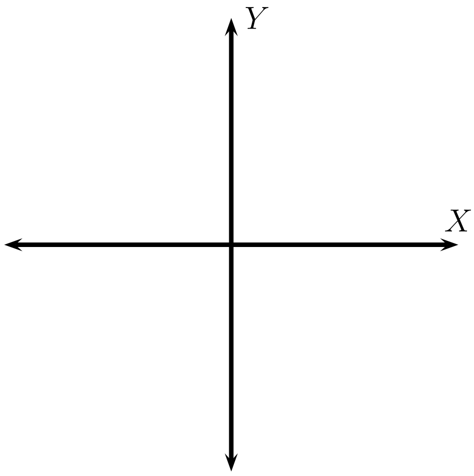
(3) $f(x) = \left(\frac{1}{2}\right)^x$



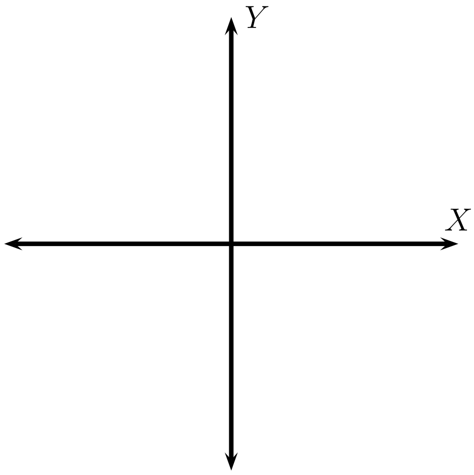
(4) $f(x) = \left(\frac{1}{3}\right)^x$



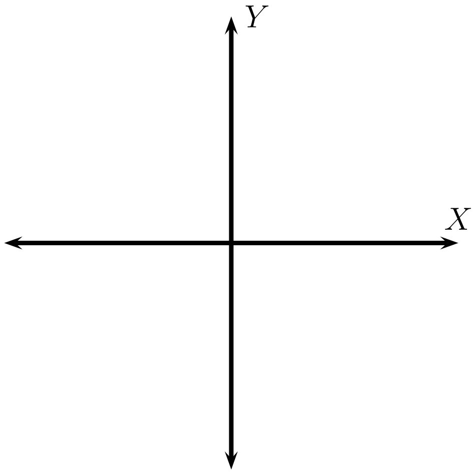
(5) $f(x) = 2^x + 1$



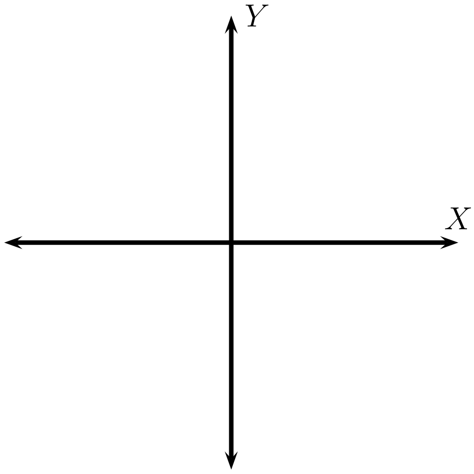
(6) $f(x) = \left(\frac{1}{3}\right)^x - 2$



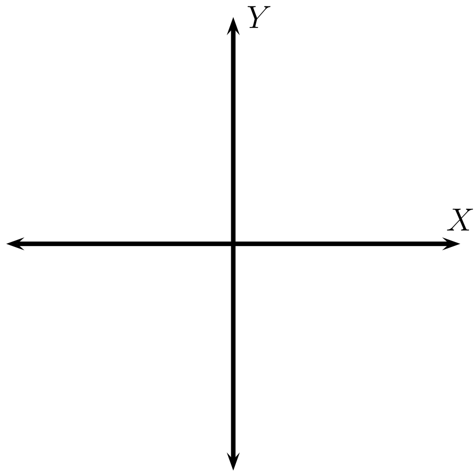
(7) $f(x) = 3 \cdot 2^x$



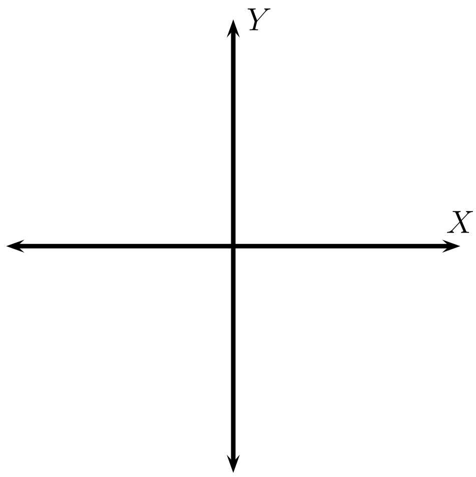
(8) $f(x) = 2^{x+1} = 2 \cdot 2^x$



(9) $f(x) = \left(\dfrac{5}{2}\right)^x$



(10) $f(x) = \left(\dfrac{2}{5}\right)^x$

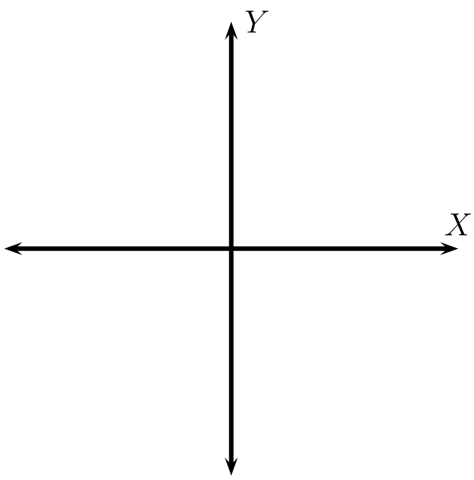


(11) Using an example discuss the graphs of $f(x) = b^x$ and $g(x) = b^{-x}$ when $b > 1$. Your discussion should include concepts of *increasing/decreasing*, *horizontal asymptotes*, and *symmetry of the two graphs.*

(12) Discuss the graphs of $f(x) = b^x$ and $g(x) = c^x$ when $b > c > 1$. Your discussion should include concepts of *steepness*, and *horizonal asymptotes*.

(13) Discuss the graphs of $f(x) = b^x$ and $g(x) = c^x$ when $0 < b < c < 1$. Your discussion should include concepts of *steepness*, and *horizonal asymptotes*.

(14) Use the graphing calculator to graph $f(x) = 0.3(1.2^{2x})$



When P dollars are deposited at the rate of interest of r per annum compounded continuously, the final amount A and the interest I earned after t number of years are given by the formulae

$$A = Pe^{rt} \quad I = A - P.$$

Find the final amount and the interest earned when \$ 250 is deposited at 6 % interest compounded continuously (do not forget to convert the % to a numeric value) for

- 1 year
- 2 years
- 15 years

5. Logarithmic Functions

The Logarithmic function is the **inverse** function of the exponential function.

For $b > 0, b \neq 1$,

$$\log_b x = y \textit{ if and only if } b^y = x.$$

What values of x is $\log_b x$ undefined? When is $\log_b x = 0$?

(1) Convert each statement to a radical equation.

(a) $2^4 = 16$.

(b) $3^5 = 243$.

(c) $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$.

(2) Convert each statement to a logarithmic equation. How is this form different from the radical form?.

(a) $2^4 = 16$.

(b) $3^5 = 243$.

(c) $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$.

(d) $\left(\frac{2}{3}\right)^{-4} = \underline{\hspace{3cm}}$.

(3) Convert each statement to exponential form:

(a) $\log_{10} 1000 = 3$.

(b) $\log_{\frac{1}{3}} 9 = -2$.

(c) $\log_{25} 5 = \frac{1}{2}$.

(d) $\log_5 25 = 2$.

Solve for x:

(1) $\log_5 x = 1$

(2) $\log_2 x = 0$

(3) $\log_x 16 = 4$

(4) $\log_x 4 = 16$

(5) $\log_3 81 = x$

(6) $\log_2 \dfrac{1}{32} = x$

(7) $\log_{36} \dfrac{1}{6} = x$

(8) $\log_x 12 = 2$

(9) $\log_x 12 = \dfrac{1}{2}$

(10) $\log_x 9 = 2$

(11) $\log_2(x-4) = 4$

(12) $\log_3 243 = (2x+3)$

(13) $\log_{125} x = \dfrac{1}{3}$

(14) $\log_5 x = \dfrac{1}{3}$

(15) $\log_{10} x = 10$

(16) $\log_5(x^2 - 5x + 1) = 2$

(17) $\log_3(6x^2 - 5x + 23) = 3$

Solve the literal equations for the given variable:

(1) $a + b = 3\log_4\left(\dfrac{x}{y}\right)$ (solve for x).

(2) $-3 = a + \log_2(3y)$ (solve for y).

6. Properties of logarithms

Recall that for $b > 0, b \neq 1$, $\log_b x = y$ if and only if $b^y = x$. Give proofs for the following:

- For $b > 0, b \neq 1$, we have $\log_b b^x = x$

- For $b > 0, b \neq 1$, we have $b^{\log_b x} = x$

- For $b, M, N > 0, b \neq 1$, we have $\log_b(M \cdot N) = \log_b M + \log_b N$.

- For $b, M, N > 0, b \neq 1$, we have $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$.

- For $b, M > 0, b \neq 1$, we have $\log_b(M^p) = p \log_b M$.

- For $b, c, M > 0, b, c \neq 1$, we have $\dfrac{\log_c M}{\log_c b} = \log_b M$.

Now use these properties to expand the following logarithmic expressions:

(1) $\log_3 15$

(2) $\log_4 64x$

(3) $\log_2(32x^2y^3)$

(4) $\log_{125}\left(\dfrac{5}{x^2}\right)$

(5) $\log_{49}\left(\dfrac{7}{x^2y^3}\right)$

(6) $\log_3(243x)$

(7) $\log_5\left(\dfrac{125}{y}\right)$

(8) $\log_7\left(\dfrac{343\sqrt{MN}}{\sqrt[3]{7}}\right)$

(9) $\log\left(\dfrac{x^5\sqrt[5]{a+b+100}}{1000(x-2)^2}\right)$

(10) $\ln \sqrt[7]{\dfrac{e^3}{4}}$

(11) $\ln(M+N)$

Use the properties of the logarithm function to condense the following expressions:

(1) $\log 25 + \log 4$

(2) $\log_3 x^4 + 2\log_3 y - 5\log_3 z$

(3) $\dfrac{1}{2}\log x + \dfrac{1}{3}\log y$

(4) $3\ln(x+y) - 4\ln(a+b) + 2\ln z - \dfrac{2}{3}\ln p$

(5) $\log_6 x + \log_6(x+3) - \log_6(x^2-9)$

(6) $\log M \cdot \log N \div \log P$

Find the exact values of the following (no calculators):

• $\log_3 81$ • $14\log_5 \sqrt[7]{5}$ • $4\log_7 \sqrt[5]{7}$

7. Natural and Common Logarithms

(1) What is the natural logarithm?

(2) What is the common logarithm?

(3) Convert the following to log-expressions (common logarithm) and then use your calculator to find their values:

(a) $\log_6 34$

(b) $\log_{12} 123$

(4) Convert the following to ln-expressions (natural logarithm) and then use your calculator to find their values:

(a) $\log_4 19.34$

(b) $\log_{11} 49$

(5) Evaluate (no calculators):

(a) $\ln(e^{23})$

(b) $\ln(\sqrt[3]{e^{18}})$

(c) $\frac{1}{3}\ln(e^{18})$

(d) $\log(1,000,000)$

(e) $\log(\sqrt[5]{1,000,000})$

(f) $\frac{1}{5}\log(1,000,000)$

(g) $\log\left(\frac{1}{1,000}\right)$

8. Exponential and Logarithmic Equations

Solve for x

(1) $2^x = 256$

(2) $3^x = 81$

(3) $4^{x-1} = 64$

(4) $5^{x+1} = \dfrac{1}{25}$

(5) $6^{x-3} = \dfrac{1}{36}$

(6) $5^{3x} = 2^{7x-1}$

(7) $3^x = 4^{x-5}$

(8) $3e^{7x-3} = e^{10x}$

(9) $4(10^{9x+2}) = 100$

(10) $3(10^{3x+2}) = 5^{4x}$

(11) $\log(3x+1) - \log 4 = \log(2x+1)$

(12) $\log(x-3) + \log 8 = \log 90$

(13) $\log(2x+5)+\log 3=\log(3x+5)$

(14) $\log(x-6)+\log(x+4)=\log 3+\log x$

(15) $2\ln(x-1)=\ln 5+\ln x-\ln 6$

9. An Introduction to Vectors

(1) What is a scalar? Give three examples of how we encounter scalars.

(2) What is a vector? Change your three examples above to have them represent vectors.

(3) Explain in your own words and illustrations how to

(a) add vectors using the polygonal method;

(b) add two vectors using the method of parallelogram;

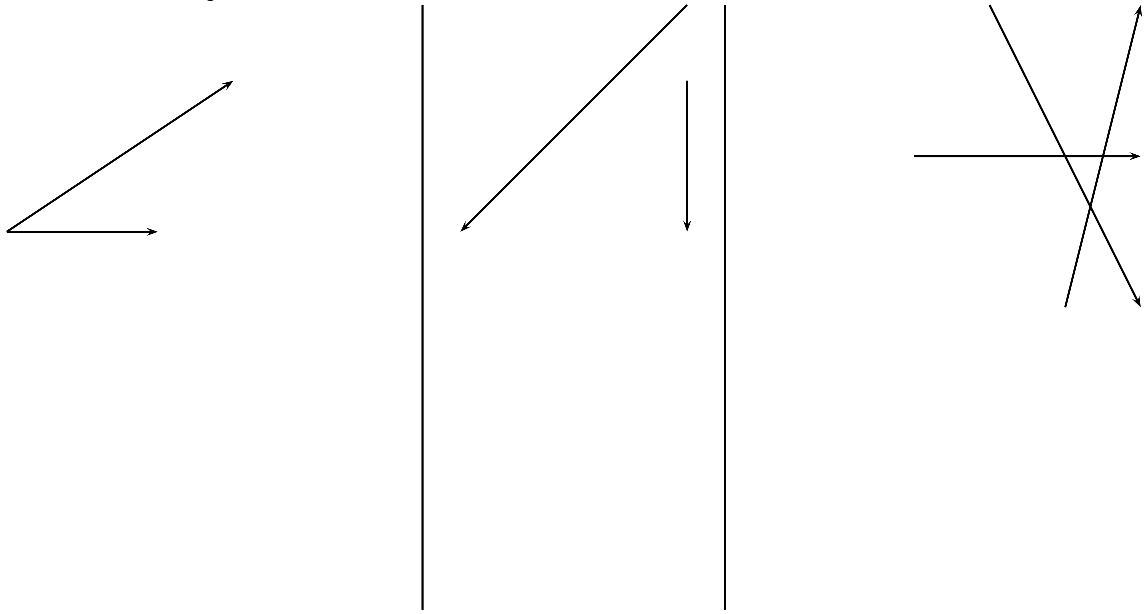
(c) find the difference of two vectors;

(d) scale a vector by 2;

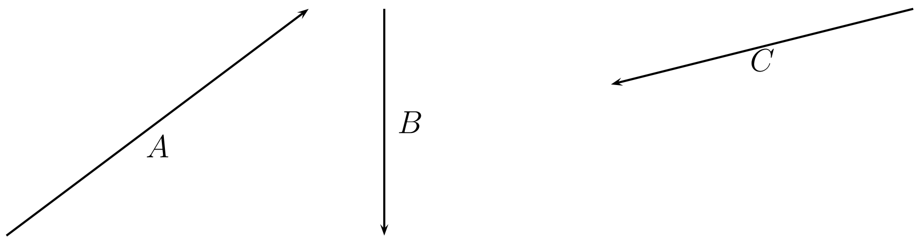
(e) scale a vector by $\dfrac{1}{3}$;

(f) draw a linear combination of two vectors.

(4) Add the following vectors and draw the resultant:



(5) Draw the indicated sums of vectors:



• $A + B + C$ • $\frac{1}{2}A - B + C$ • $A + 2B - 3C$

(6) Draw the following vectors and draw their sums.

(a) 2.3 cm. 15^o; 3.2 cm. 120^o.

(b) 1.2 cm. 45^o; 2.3 cm. 0^o.

10. Components of Vectors

(1) What are the components of a vector? Explain with an example and illustration.

(2) What are the x- and y- components of a vector on a coordinate plane? Explain with an example and illustration.

(3) On a coordinate plane draw the vector starting at origin and ending at the point $(3, 2)$. Resolve this vector into its x and y components. That is, which points determine the x and y components of this vector?

(4) On a coordinate plane draw the vector starting at origin and ending at the point $(-3, 2)$. Resolve this vector into its x and y components.

(5) On a coordinate plane draw the vector starting at origin and ending at the point $(3, -2)$. Resolve this vector into its x and y components.

(6) On a coordinate plane draw the vector starting at origin and ending at the point $(-3, -2)$. Resolve this vector into its x and y components.

(7) On a coordinate plane draw the vector starting at origin with magnitude 2.1 cm and direction $\theta = 132^o$. Resolve this vector into its x and y components. (Use calculator).

(8) On a coordinate plane draw the vector starting at origin with magnitude 1.3 cm and direction $\theta = 235^o$. Resolve this vector into its x and y components. (Use calculator).

11. Vector Addition by Components

(1) Draw illustrations and add the vectors **A** and **B** by components where (don't forget the resultant direction)

(a) $A = 23, \theta_A = 35^o, B = 35, \theta_B = 78^o$.

(b) $A = 23, \theta_A = 105^o, B = 35, \theta_B = 78^o$.

(c) $A = 23, \theta_A = 115^o, B = 35, \theta_B = 128^o$.

(d) $A = 23, \theta_A = 215^o, B = 35, \theta_B = 78^o$.

(2) A child throws a paper plane towards the east at the speed of 0.5 mi/hr. At the same time, wind blows from the south towards the north at the speed of 3 mi/hr. What is the velocity of the paper plane?

(3) A man is rowing a boat on the river going north at the speed of 1.5 mi/hr. The current of the river is pushing the boat west at the speed of 1.8 mi/hr. What is the resultant velocity of the boat?

12. Applications of Vectors

Go through as many problems from the textbook as time permits.

13. Complex Numbers: Basic Definitions

The **imaginary unit** is

$$j = \sqrt{-1}; \quad j^2 = -1.$$

We are following the notation used by our textbook. Most authors use the letter i to represent the imaginary unit.

Evaluate:

- $\sqrt{81}$
- $\sqrt{-81}$
- $-\sqrt{81}$
- $\sqrt{25}$
- $\sqrt{-25}$
- $-\sqrt{25}$
- $\sqrt{36}$
- $\sqrt{-36}$
- $-\sqrt{36}$
- $-\sqrt{121}$
- $\sqrt{-121}$
- $-\sqrt{-121}$
- $-\sqrt{1}$
- $\sqrt{-1}$
- $-\sqrt{-1}$
- $-\sqrt{100}$
- $\sqrt{-100}$
- $-\sqrt{-100}$
- $-\sqrt{75}$
- $\sqrt{-75}$
- $-\sqrt{-75}$
- $-3\sqrt{8}$
- $3\sqrt{-8}$
- $-3\sqrt{-8}$
- $\sqrt{24}$
- $\sqrt{-24}$
- $-\sqrt{-24}$
- $\sqrt{-7}$
- $(\sqrt{-7})^2$
- $\sqrt{(-7)^2}$
- $\sqrt{-12}$
- $(\sqrt{-12})^2$
- $\sqrt{(-12)^2}$
- $\sqrt{(-3)(-4)}$
- $\sqrt{(-2)(-6)(-3)}$
- $\sqrt{(-6)(-8)}$
- $\sqrt{(-3)}\sqrt{(-4)}$
- $\sqrt{(-2)}\sqrt{(-6)}\sqrt{(-3)}$
- $\sqrt{(-6)}\sqrt{(-8)}$
- j^2
- j^3
- j^4
- j^5
- j^6
- j^7
- j^8
- j^9
- j^{10}
- j^{15}
- j^{26}
- j^{37}
- j^{-2}
- j^{-3}
- j^{-4}
- j^{-5}
- j^{-6}
- j^{-7}
- j^{-8}
- j^{-9}
- j^{-10}
- j^{-15}
- j^{-26}
- j^{-37}

14. Basic Operations with Complex Numbers

Evaluate:

(1) $(5+4j)-(3+7j)$

(2) $(-5+4j)+(3-7j)$

(3) $(30-15j)-(2+5j)+(-8+3j)$

(4) $(30-15j)-[(2+5j)+(-8+3j)]$

(5) $\left(\frac{2}{3}+\frac{5}{4}j\right)-\left(\frac{4}{7}-\frac{8}{3}j\right)-\left(\frac{1}{42}+\frac{j}{12}\right)$

(6) $\left(\frac{2}{3}+\frac{5}{4}j\right)-\left[\left(\frac{4}{7}-\frac{8}{3}j\right)-\left(\frac{1}{42}+\frac{j}{12}\right)\right]$

(7) $5(3+4j)$

(8) $-5(3+4j)$

(9) $5j(3+4j)$

(10) $(1+5j)(3+4j)$

(11) $(-1-2j)(-3-4j)$

(12) $(4-5j)(5+12j)$

(13) $j(3-4j)$

(14) $(4+5j)^2$

(15) $(4-5j)^2$

(16) $(4+5j)^3$

(17) $(4-5j)^3$

(18) $(4+5j)(4-5j)$

(19) $\left(\dfrac{1}{4}+\dfrac{2}{5}j\right)\left(\dfrac{4}{5}-\dfrac{3}{7}j\right)$

(20) $\dfrac{3+4j}{5}$

(21) $\dfrac{3+4j}{-5}$

(22) $\dfrac{3+4j}{5j}$

(23) $\dfrac{3+4j}{-5j}$

(24) $\dfrac{3+4j}{1+5j}$

(25) $\dfrac{-4-3j}{-1-2j}$

(26) $\dfrac{5+12j}{4-5j}$

(27) $\dfrac{j}{3-4j}$

(28) $\dfrac{1}{3-4j}$

(29) $\dfrac{1}{5+12j}$

(30) $\dfrac{1}{-3-4j}$

(31) You have encountered complex conjugates above. Explain what they are using examples.

15. Graphical Representation of Complex Numbers

(1) Locate the given numbers on the complex plane:

- $2 + 3j$
- $3 - 2j$
- $-2 - j$
- $-2 + 4j$

(2) Add/Subtract the following numbers graphically:

$(2 + 3j) + (-3 + 4j)$

$(2 + 3j) - (1 - 3j)$

(3) Show the given number, its negative, and its conjugate on the same complex plane: $4+3j$

(4) Show the given number, its negative, and its conjugate on the same complex plane: $-2+3j$

(5) Show the given number, its negative, and its conjugate on the same complex plane: $6-j$

16. Polar Form of a Complex Number

Relationship between the rectangular coodinates and the polar coordinates:

$$x = r\cos\theta; \quad y = r\sin\theta; \quad r^2 = x^2 + y^2; \quad \tan\theta = \frac{y}{x}.$$

Therefore, $x + yj = r(\cos\theta + j\sin\theta)$. An important notation:

$$r\angle\theta = r(\cos\theta + j\sin\theta).$$

(1) Represent each complex number graphically and give the polar form of each:

(a) $3 - 4j$

(b) $-4j$

(c) 5

(d) $1.3 + 2j$

(2) Represent each complex number graphically and give the rectangular form of each:

(a) $3.0(\cos 38^o + j \sin 38^o)$

(b) $2.3(\cos 138^o + j \sin 138^o)$

(c) $2.5\angle - 200^o$

(d) $1.2\angle 0^o$

17. Exponential Form of a Complex Number

The **exponential form** is $re^{j\theta} = r(\cos\theta + j\sin\theta)$ where θ is measured in **radians**.

Recall, π radians $= 180^o$.

Summary of relationships between rectangular, polar, and exponential forms of a complex number:

Rectangular form:	$x + jy;$
Polar form:	$r(\cos\theta + j\sin\theta);$
Exponential form:	$re^{j\theta}.$

Further, $r^2 = x^2 + y^2$ and $\tan\theta = \dfrac{y}{x}$.

(1) Convert the following measurements of angles from degrees to radians:

•210^o • 40^o • 130^o • 290^o

(2) Convert the following measurements of angles from radians to degrees:

•2.57 *rad* • 4.02 *rad* • 1.30 *rad* • 5.9 *rad*

(3) Express the given complex numbers in polar form and in rectangular form:

(a) $2e^{2.34j}$

(b) $1.3e^{5.89j}$

(c) $0.8e^{0j}$

(d) $1.2e^{\frac{\pi}{2}j}$

(4) Express the given complex numbers in polar form and in exponential form:

(a) $2 + 3j$

(b) $-3 + 2j$

(c) $-4 - 3j$

(d) $3-4j$

(e) $4j$

(f) 3

18. Products, Quotients, Powers, and Roots

(1) Explain how to find the product of two complex numbers written in their exponential form or polar form. Give three examples.

(2) Explain how to find the quotient of two complex numbers written in their exponential form or polar form. Give three examples.

(3) State DeMoivre's theorem. Use this theorem to explain how to find a power of a complex number written in its exponential form or polar form. Give three examples.

(4) Explain how to find both the two square-roots of a complex number written in its exponential form or polar form. Give three examples.

(5) Explain how to find all the three cube-roots of a complex number written in its exponential form or polar form. Give three examples.

(6) First convert the given complex numbers to their exponential form, and then perform the indicated operation. Finally, write your results in their rectangular form.

(a) $(2 + 3j)(-4 + j)$

(b) $\dfrac{1 - 2j}{3 - 2j}$

(c) $(4-3j)^5$

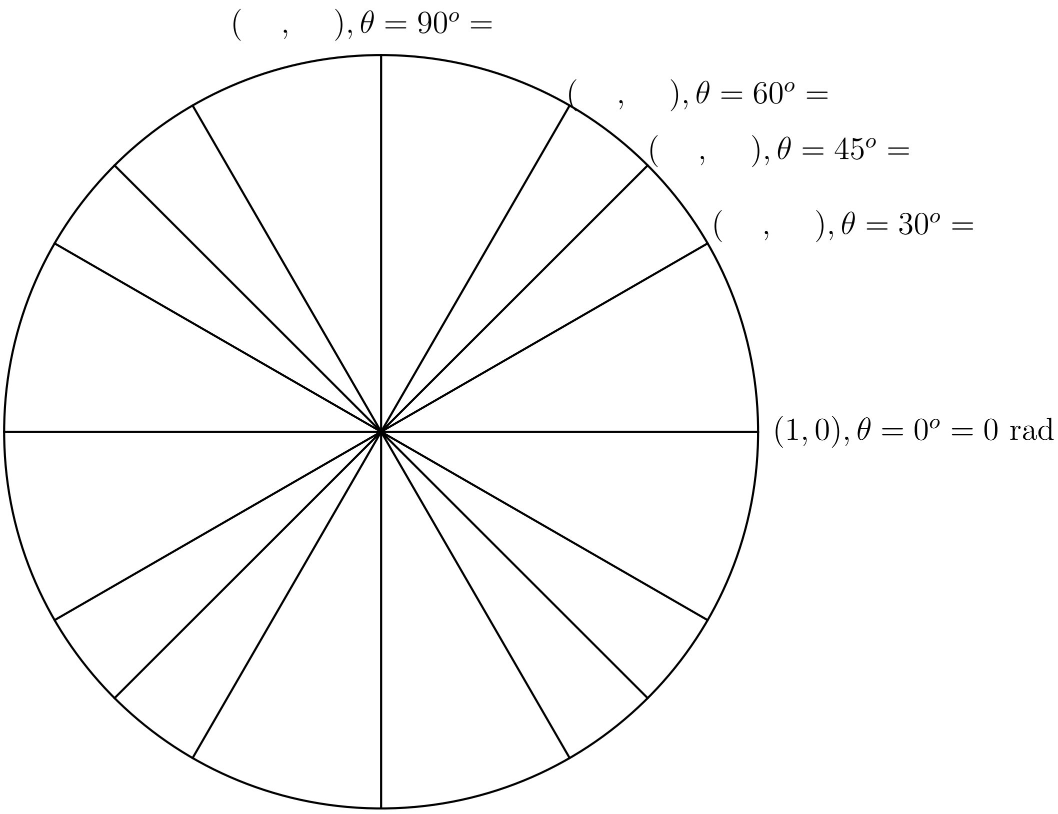
(d) Find both the square-roots of $3-4j$.

(e) Find all the three cube-roots of $2-3j$.

(f) Find all the four fourth-roots of $-3-2j$.

19. Graphs of $y = a \sin x$ and $y = a \cos x$

The unit circle in the coordinate plane is the circle of radius 1 centered at $(0,0)$. For a point (x,y) on the unit circle, we have $x = \cos\theta$ and $y = \sin\theta$ where θ is the angle subtended by the ray originating from $(0,0)$ to (x,y) with respect to the X-axis. Use calculators or your knowledge of trignometry to find rectangular coordinates for all the end points of the radial segments shown on the unit circle below. Give the angles in both degree and radian form.



Here is a way of remembering the numbers you derived above:

$$0 < 1 < 2 < 3 < 4$$

Take square root throughout

Divide throughout by 2

How are these numbers to be used?

(1) Plot the graph of $y = \sin x$, where x is measured in radians. Use values

$x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}$.

(2) Plot the graph of $y = \cos x$, where x is measured in radians. Use values

$x = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}$.

(3) Plot the graph of $y = 3\sin x$, where x is measured in radians.

(4) Plot the graph of $y = 2\cos x$, where x is measured in radians.

(5) Plot the graph of $y = -2\sin x$, where x is measured in radians.

(6) Plot the graph of $y = -3\cos x$, where x is measured in radians.

(7) Find the function $y = a \sin x$ which passes through the point $(\frac{\pi}{2}, 4)$ and graph it.

(8) Find the function $y = a \cos x$ which passes through the point $(\pi, -2)$ and graph it.

(9) Define the amplitude for a sine curve or a cosine curve.

20. Graphs of $y = a\sin bx$ and $y = a\cos bx$

(1) What is the period for a sine curve or a cosine curve? Give three examples.

(2) Find the period of the function $y = \sin(3x)$ and graph it.

(3) Find the period of the function $y = \cos(4x)$ and graph it.

(4) Find the amplitude and the period of the function $y = 3\sin(2x)$ and graph it.

(5) Find the amplitude and the period of the function $y = -2\cos(3x)$ and graph it.

(6) Find the amplitude and the period of the function $y = 2\sin(\frac{x}{3})$ and graph it.

(7) Find the amplitude and the period of the function $y = -\cos(\frac{x}{3})$ and graph it.

(8) Find the amplitude and the period of the function $y = 3\sin(2\pi x)$ and graph it.

(9) Find the amplitude and the period of the function $y = \frac{2}{3}\cos(5\pi x)$ and graph it.

(10) Find the function $y = -3\sin(bx)$ whose curve passes through $(1, 3)$ and b has the smallest possible value.

(11) Find the function $y = \frac{2}{3}\cos(bx)$ whose curve passes through $(3, 0)$ and b has the smallest possible value.

21. Graphs of $y = a\sin(bx + c)$ and $y = a\cos(bx + c)$

(1) What is the displacement or phase-shift for a sine or a cosine curve. Give three examples. (Hint: Set $bx + c = 0$).

(2) Determine the amplitude, the period, and the phase-shift of the function $y = \sin(x + \pi)$ and graph it.

(3) Determine the amplitude, the period, and the phase-shift of the function $y = \cos(x - \pi)$ and graph it.

(4) Determine the amplitude, the period, and the phase-shift of the function $y = 3\sin(x + \frac{\pi}{3})$ and graph it.

(5) Determine the amplitude, the period, and the phase-shift of the function $y = -3\cos(x - \frac{\pi}{6})$ and graph it.

(6) Determine the amplitude, the period, and the phase-shift of the function $y = -3\sin(2x+\pi)$ and graph it.

(7) Determine the amplitude, the period, and the phase-shift of the function $y = 3\cos(2x-\pi)$ and graph it.

22. Graphs of $y = \tan x,\ y = \cot x,\ y = \sec x,\ y = \csc x$

$$\tan x = \frac{\sin x}{\cos x}; \quad \cot x = \frac{\cos x}{\sin x}; \quad \sec x = \frac{1}{\cos x}; \quad \csc x = \frac{1}{\sin x}.$$

(1) Graph one cycle of $y = \tan x$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(2) Graph one cycle of $y = \cot x$, $0 < x < \pi$.

(3) Graph one cycle of $y = \sec x$, $0 < x < 2\pi$, indicate the vertical asymptotes.

(4) Graph one cycle of $y = \csc x$, $0 < x < 2\pi$, indicate the vertical asymptotes.

23. Applications of the Trignometric Graphs

Try out as many problems from the textbook as time permits.

24. Fundamental Trignometric Identities

Recall from your previous courses in mathematics how trignometric functions are first defined.

(1) Let point (x, y) be on a circle centered at $(0, 0)$ with radius r. Assume that $x, y > 0$. Let θ denote the positive angle formed by the ray starting at $(0, 0)$ and passing through (x, y). Use your illustration to prove that $\sin\theta = \dfrac{y}{r}$, $\cos\theta = \dfrac{x}{r}$, and $\tan\theta = \dfrac{y}{x}$.

(2) Extend the definitions above to all real number x, y. Note, the tan function will not be defined for values $x = 0$. Now, $\sec\theta = \dfrac{1}{\cos\theta}$, $\csc\theta = \dfrac{1}{\sin\theta}$, and $\cot\theta = \dfrac{1}{\tan\theta}$. Using these relations, show that

$$\sin^2\theta + \cos^2\theta = 1; \quad 1 + \tan^2\theta = \sec^2\theta; \quad 1 + \cot^2\theta = \csc^2\theta.$$

(3) Simplify the following expressions reproduced here from the textbook.

- $\tan y(\cot y + 3\cos y)$

- $(\csc x - 1)(\csc x + 1)$

- $\tan^2 u \sec^2 u - \tan^4 u$

- $\dfrac{1 + \tan x}{\sin x} - \sec x$

(4) Prove the following identities reproduced here from the textbook.

- $\sec\theta(1 - \sin^2\theta) = \cos\theta$

- $\csc x \sec x - \tan x = \cot x$

- $\dfrac{1+\cos x}{\sin x} = \dfrac{\sin x}{1-\cos x}$

- $\dfrac{\sec\theta}{\cos\theta} - \dfrac{\tan\theta}{\cot\theta} = 1$

25. The Sum and Difference Formula

(1) Using the definition of $\sin\theta$ and $\cos\theta$ derived from a point on a circle, show that $\sin(-\theta) = -\sin(\theta)$ and $\cos(-\theta) = -\cos(\theta)$.

(2) Using exponential form of complex numbers, prove that

$$\sin(\alpha+\beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta); \quad \cos(\alpha+\beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta).$$

(3) Prove that $\sin(\alpha-\beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$.

(4) Prove that $\cos(\alpha-\beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$.

(5) Prove that $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$.

(6) Prove that $\tan(\alpha-\beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$.

(7) Evaluate $\sin(75^o)$ by using $75^o = 30^o + 45^o$.

(8) Evaluate $\cos(75^o)$ by using $75^o = 30^o + 45^o$.

(9) Evaluate $\tan(75^o)$ by using $75^o = 30^o + 45^o$.

(10) Evaluate $\sin(15^o)$ by using $15^o = 45^o - 35^o$.

(11) Evaluate $\cos(15^o)$ by using $15^o = 45^o - 35^o$.

(12) Evaluate $\tan(15^o)$ by using $15^o = 45^o - 35^o$.

26. Double-angle Formulae

(1) State and prove the sine double-angle formula.

(2) State and prove the cosine double-angle formula.

(3) State and prove the tangent double-angle formula.

(4) Prove the identity: $\cos(2\theta) = 1 - 2\sin^2(\theta) = 2\cos^2(\theta) - 1$.

(5) Find $\sin(120^o)$ using sin and cos values for 60^o.

(6) Find $\cos(120^o)$ using sin and cos values for 60^o.

(7) Find $\tan(120^o)$ using tan value for 60^o.

(8) Find $\tan(2x)$ if $\sin(x) = 0.3$ and the terminal point for angle x radians is in the second quadrant.

(9) Simplify the expression reproduced from the textbook: $\dfrac{\cos(3x)}{\sin(x)} + \dfrac{\sin(3x)}{\cos(x)}$.

(10) Prove the identity reproduced from the textbook: $\dfrac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} = 1 - \dfrac{1}{2}\sin 2\theta$.

27. Half-angle Formulae

(1) Use the identity $\cos 2\theta = 1 - 2\sin^2\theta$ to prove the half-angle formula $\sin\frac{\alpha}{2} =$.

(2) Use the identity $\cos 2\theta = 2\cos^2\theta - 1$ to prove the half-angle formula $\cos\frac{\alpha}{2} =$.

(3) Use the half-angle formula to find $\sin(22.5^o)$.

(4) Use the half-angle formula to find $\cos(22.5^o)$.

(5) Simplify the expression: $\sqrt{2 + 2\cos(8x)}$.

(6) Simplify the expression: $\sqrt{1 - \cos(120^o)}$

(7) Given that $\tan\theta = -\dfrac{7}{9}$ and $90^o < \alpha < 180^o$ find $\sin(\frac{\alpha}{2})$.

(8) Simplify the expression reproduced from the textbook: $2\sin^2(\frac{x}{2}) + \cos(x)$.

(9) Prove the identity reproduced from the textbook: $\cos(\frac{\theta}{2}) = \dfrac{\sin(\theta)}{2\sin(\frac{\theta}{2})}$

28. Solving Trignometric Equations

Some of the following equations have been reproduced from the textbook. Solve for x, where $0 \le x < 2\pi$:

(1) $2\sin x - 1 = 0$

(2) $2\sin^2 x - 1 = 0$

(3) $\sin(x) = \cos(x)$

(4) $\sin(x - \frac{\pi}{4}) = \cos(x - \frac{\pi}{4})$

(5) $2\cos^2 x - \cos x = 0$

(6) $2\sin^2 x - 3\sin x + 1 - 0$

(7) $\tan^2 x + 6 = 5\tan x$

(8) $\sin x \sin \frac{1}{2}x = 1 - \cos x$

(9) $\cos 3x \cos x - \sin 3x \sin x = 0$

(10) $2\sin 2x - \cos x \sin^3 x = 0$

29. The Inverse Trignometric Functions

The sine function is one-to-one on the domain $[-\frac{\pi}{2}, \frac{\pi}{2}]$. That is, on this domain, the graph of the sine function passes the **horizontal line test**. Therefore, this function has an inverse, the **inverse sine function** denoted by $\sin^{-1}$.

$$y = \sin^{-1} x \quad \text{if and only if} \quad \sin y = x, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

Notice that the $\sin^{-1} x$ is defined for $-1 \leq x \leq 1$. That is, the domain of the inverse sine function is $[-1, 1]$ and its range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

Similarly, the cosine function is one-to-one on the domain $[0, \pi]$. Thus, the **inverse cosine function**, denoted by $\cos^{-1}$ is defined on the set $[-1, 1]$ with range $[0, \pi]$ by

$$y = \cos^{-1} x \quad \text{if and only if} \quad \cos y = x, 0 \leq y \leq \pi.$$

Lastly, the tangent function is one-to-one on the domain $(-\frac{\pi}{2}, \frac{\pi}{2})$ with range $(-\infty, \infty)$. Hence, the **inverse tan function**, denoted by $\tan^{-1}$ is defined on the set $(-\infty, \infty)$ with range $(-\frac{\pi}{2}, \frac{\pi}{2})$ by

$$y = \tan^{-1} x \quad \text{if and only if} \quad \tan y = x, -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

(1) Find
- $\sin^{-1} 1$
- $\cos^{-1}(-\frac{\sqrt{2}}{2})$
- $\sin(\tan^{-1} 2)$
- $\sin^{-1} 2$
- $\tan(\sin^{-1} 1)$

(2) Use calculator to find (use your calculator in radian mode):
- $\sin^{-1}(0.34)$
- $\cos \sin^{-1}(0.8)$

(3) Draw graphs of the functions $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ below:

30. Systems of Linear Equations and Cramer's Method

(1) Consider the system of equations $\begin{aligned} x+y &= 1 \\ 2x+y &= -1 \end{aligned}$. Solve this system by any method.

(2) What is a 2×2 matrix? Give three examples.

(3) Give formula for the determinant of a 2×2 matrix. Find the determinant of the matrices you gave above.

(4) Use Cramer's method to solve the system of equations given in the first question above.

(5) When does the Cramer's method fail to produce a solution? Give an example.

(6) Solve these problems reproduced from the textbook using Cramer's method:

- $$\begin{aligned} x + 3y &= 7 \\ 2x + 3y &= 5 \end{aligned}$$
- $$\begin{aligned} 2x + y &= 4 \\ 19x = 80 & -10y \end{aligned}$$
- $$\begin{aligned} 3i_1 + 5 &= -4i_2 \\ 3i_2 = 5i_1 & -2 \end{aligned}$$
- $$\begin{aligned} 0.06x + 0.048y &= -0.084 \\ 0.013x - 0.065y &= -0.078 \end{aligned}$$

(7) What is a 3×3 matrix? Give three examples.

(8) Give a formula for the determinant of a 3×3 matrix. Find the determinants of the three matrices you gave above.

(9) Use Cramer's method to solve the following systems reproduced here from the textbook:

- $$\begin{aligned} 4x + y + z &= 2 \\ 2x - y - z &= 4 \\ 3y + z &= 2 \end{aligned}$$

- $$\begin{aligned} 3r + s - t &= 2 \\ r - 2s + t &= 0 \\ 4r - s + t &= 3 \end{aligned}$$

31. Practice Problems

The following are questions used in quizzes and exams of MTH 13 in previous semesters.

(1) Perform indicated operation and write the result in rectagular form.

a) $(3+7i)-3(-2i+6)$ b) $\dfrac{-2+5i}{1-2i}$ c) $\sqrt{-16}(-2+\sqrt{-36i^2})$

(2) Find the all values of x and y that satisfy the equation

a) $(x-1)+yi=3-y^2i$ b) $x-2i^2+7i=yj+2xi^3$

(3) For each given complex number fill in the blank.

rectangular form	polar form	exponential form
$-2+i\sqrt{5}$		
	$2\angle 135°$	

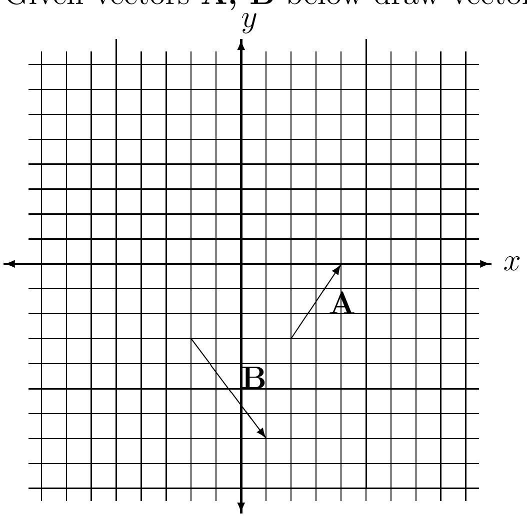
(4) Perform each operation.

a) $(-2i^2-3i^{11})(2-3i)$ b) $\dfrac{12\angle 70°}{128\angle 120°}\cdot(2\angle 10°)^5$ c) $\dfrac{8(\cos 100°+i\sin 100°)}{2(\cos 40°+i\sin 40°)}$

(5) a) Find the exponential form of the complex number $-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}$

b) Evaluate $\left(-\dfrac{1}{2}+i\dfrac{\sqrt{3}}{2}\right)^6$ c) Find two square roots of $\dfrac{1}{2}-i\dfrac{\sqrt{3}}{2}$.

(6) Given vectors **A, B** below draw vector **2A−B**



(7) Find the x-component A_x and y-components A_y of each vector.

a) $\mathbf{A}=4.85\angle 78.24°$ b) $\mathbf{A}=20.15\angle 280°$

(8) Find both the magnitude and the angle of each vector.

a) $A_x=-3, A_y=-4$ b) $B_x=-6, B_y=1$

(9) Find the resultant vector $\mathbf{A}+\mathbf{B}$ if $\mathbf{A}=6.89\angle 123.0°$ and $\mathbf{B}=7.83\angle 260.0°$.

(10) To avoid a storm, a plane flies 145 mi at 35° south of west, and 240 mi at 15° west of north. What is the displacement of the plane from its original position?

(11) In lifting a heavy piece of equipment from the mud, cable from a crane exert a vertical force of 6500 N, and a cable from a truck exerts a force of 8300 N at 10.0° above the horizontal. Find the resultant of these forces.

(12) Find the values of x and y that satisfy the equation $x - 2i^2 + 7i = yi + 3i^2$.

(13) For each given complex number fill in the blank. Give answers in exact values whenever possible.

rectangular form	polar form	exponential form
$2 + \sqrt{5}i$		
		$6e^{\frac{3\pi i}{4}}$

(14) Perform the indicated operations. Express all answers in the form $a + ib$

a) $5 + \sqrt{-\frac{16}{25}} - \sqrt{-25} + 3$ b) $(2 - i)(3 + i5)$ c) $\dfrac{2\angle 65°}{6\angle 20°}$

d) $(\sqrt[5]{3}(\cos 12° + i \sin 12°))^{15}$ e) $3i^{11} - 2i^2$ f) $\dfrac{1 + i}{4 - i3}$

(15) Fill in the blank to make an identity

a) $\sin(x - y) =$ ________________________ .

b) $\sin \dfrac{x}{2} =$ ________________________ .

c) $\cos 2x =$________________________ .

d) $\sin 2x =$ ________________________ .

e) $\tan(x + y) =$ ________________________ .

(16) Find $\cos(x + y)$ if $\sin x = \frac{2}{3}$ and x is in the second quadrant and $\sin y = -\frac{1}{3}$ and y is in the fourth quadrant.

(17) Find the exact value of $\cos 75°$. No decimal value will be accepted as an answer.

(18) Prove each identity.

a) $\sin(30° - x) + \cos(60° - x) = \cos x$ b) $(\csc x - \cot x)(\csc x + \cot x) = 1$

c) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$ d) $\dfrac{\cos 2x}{\cos^2 x} = 1 - \tan^2 x$

(19) Solve each equation for all values in $0 \leq x < 2\pi$.

a) $2 \cos x - 1 = 0$ b) $-3 \cos x + 3 = 2 \sin^2 x$.

(20) Evaluate each determinant. Simplify the result.

a) $\begin{vmatrix} \sin\theta & \cos\theta \\ \csc\theta & \sec\theta \end{vmatrix}$ b) $\begin{vmatrix} \tan\theta & -\tan\theta \\ \tan\theta & \cot\theta \end{vmatrix}$ c) $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

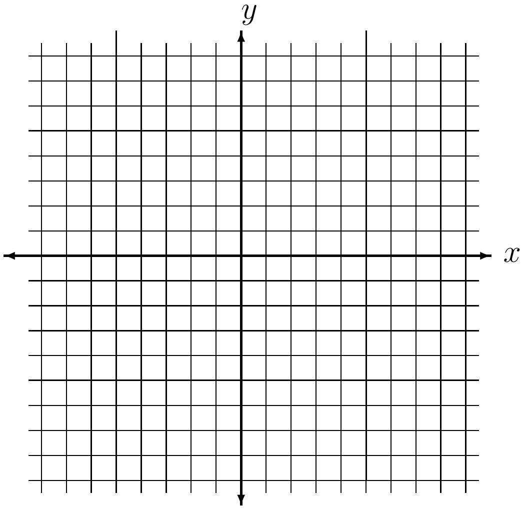
(21) Solve each system of linear equation using determinants (or Cramer's rule.)

a) $\begin{cases} 2x - 2y = 2 \\ -2x + y = 3 \end{cases}$ b) $\begin{cases} 3x + y = 5 \\ 2y - 3z = -5 \\ x + 2z = 7 \end{cases}$

(22) Find the domain of each function

a) $y = \sqrt{-3x+5}$ b) $y = \dfrac{11}{x^2+3x-18}$ c) $y = 5^x + 5$

(23) Graph both the exponential function $f(x) = 2^x$ and the logarithmic function $g(x) = \log_2 x$ on the same set of axes below. Label at least three points on each graph.



(24) Let $f(x) = x^2 + 3$ and $g(x) = x + 1$. Find

a) $f(3)$ b) $\dfrac{f(-1)}{g(-1)}$ c) $f(x+1)$

(25) Solve for x.

a) $5^{x-1} = 2$ b) $2\log(x-5) = 0$ b) $2\log_3 x - \log_3 4x = 1$

d) $5^{x^2} = 25^{x+4}$ e) $\log_x 4 + \log_x 2 = 3$

(26) Write as a single logarithm.

a) $2\log x - 3\log y$ b) $\frac{1}{3}(\log x + \log y) - \frac{1}{2}\log z$

(27) Write each logarithm as the product, sum, difference of two or more logarthims.

a) $\log_2\left(\dfrac{xy^4}{z}\right)$ b) $\log\sqrt{\dfrac{x}{y^2}}$

(28) Evaluate each determinant. Simplify the result.

a) $\begin{vmatrix} \sin\theta & \cos\theta \\ \csc\theta & \sec\theta \end{vmatrix}$ b) $\begin{vmatrix} \tan\theta & -\tan\theta \\ \tan\theta & \cot\theta \end{vmatrix}$ c) $\begin{vmatrix} e^0 & e^{\sin^2 x} \\ e^{\cos^2 x} & e \end{vmatrix}$ d) $\begin{vmatrix} \sqrt{2} & 2 \\ \sqrt{6} & \sqrt{3} \end{vmatrix}$

e) $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$ f) $\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 8 & 9 & 4 \end{vmatrix}$

(29) Solve each system of linear equation using determinants (or Cramer's rule.)

a) $\begin{cases} 2x - 2y = 2 \\ -2x + y = 3 \end{cases}$ b) $\begin{cases} 3x + y = 5 \\ 2y - 3z = -5 \\ x + 2z = 7 \end{cases}$

(30) Fill in the blank to make an identity

a) $\sin(x - y) =$ ____________________ .

b) $\sin \frac{x}{2} =$ ____________________ .

c) $\cos 2x =$ ____________________ .

d) $\sin 2x =$ ____________________ .

e) $\tan(x + y) =$ ____________________ .

(31) Find $\cos(x + y)$ if $\sin x = \frac{2}{3}$ and x is in the second quadrant and $\sin y = -\frac{1}{3}$ and y is in the fourth quadrant.

(32) Find the exact value of $\cos 75°$. No decimal value will be accepted as an answer.

(33) Prove each identity.

a) $\sin(30° - x) + \cos(60° - x) = \cos x$ b) $(\csc x - \cot x)(\csc x + \cot x) = 1$

c) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$ d) $\dfrac{\cos 2x}{\cos^2 x} = 1 - \tan^2 x$

(34) Solve each equation for all values in $0 \le x < 2\pi$.

a) $2\cos x - 1 = 0$ b) $2\sec x = 3 - \cos x$ c) $-3\cos x + 3 = 2\sin^2 x$.

(35) Prove each identity.

a) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$ b) $\dfrac{1 - \tan x}{1 + \tan x} = \dfrac{\cos 2x}{1 + \sin 2x}$

(36) Fill in the blank to make an identity

a) $\sin x \cos y - \cos x \sin y =$ ____________ .

b) $\sin \frac{x}{2} =$ ____________________ .

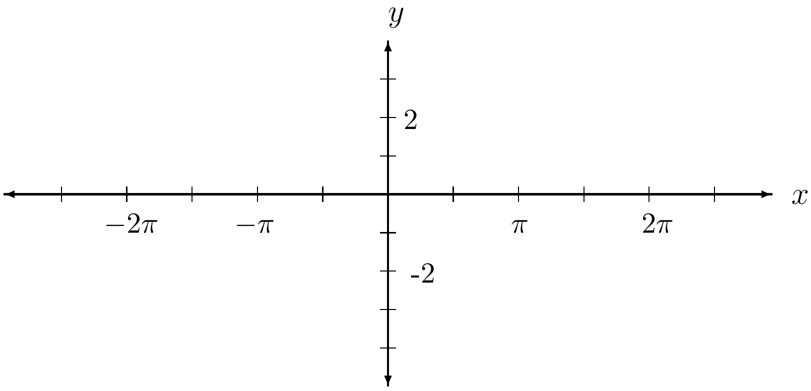
c) $\cos 2x =$ ____________________ .

(37) Find $\sin 2x$ if $\sin x = \frac{2}{3}$ and x is in the second quadrant.

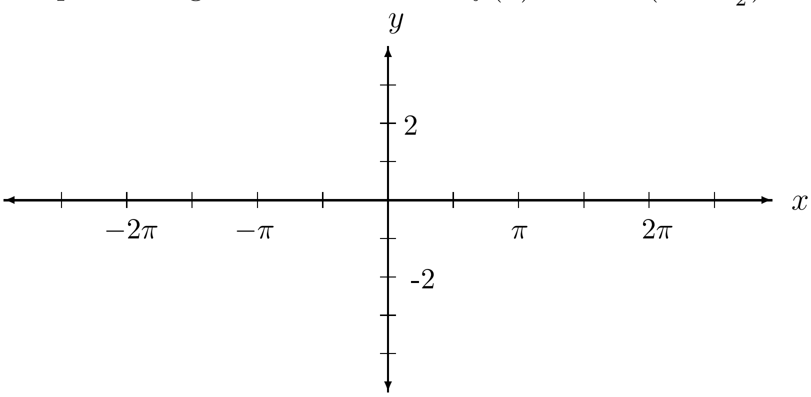
(38) Fill in the blank.

Function	Amplitude	Period	Frequency	Phase shift
$y = -3\sin(3\pi x - 2)$				
$y = 5\cos(3x - 2\pi)$				
$y = \tan(\pi x - \frac{\pi}{2})$				

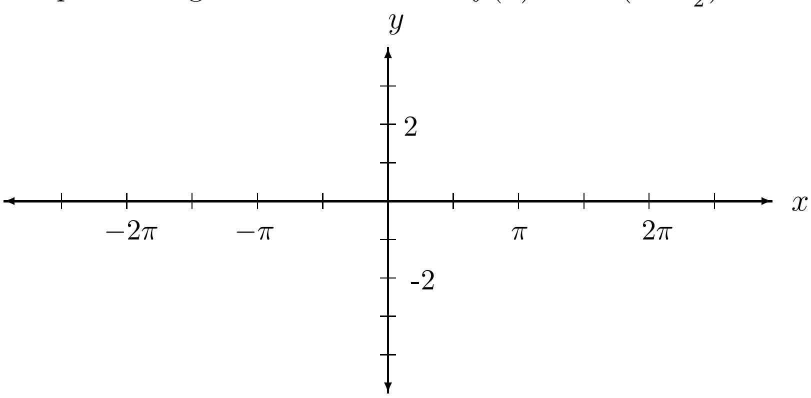
(39) Graph the trigonometric function $f(x) = 2\sin(\frac{1}{2}x - \frac{\pi}{2})$ for $-2\pi \le x \le 2\pi$.



(40) Graph the trigonometric function $f(x) = -\cos(2x - \frac{\pi}{2})$ for $0 \le x \le 2\pi$.



(41) Graph the trigonometric function $f(x) = \tan(x - \frac{\pi}{2})$ for $-\pi \le x \le \pi$.



(42) Find the smallest positive b, c of the function $y = 3\cos(bx + c)$ such that the period is 4 and phase shift is $-\frac{2}{\pi}$.

(43) Let $f(t) = 5\sin(2.34t + c)$ represents the displacement of rotating object passing through (2,0).

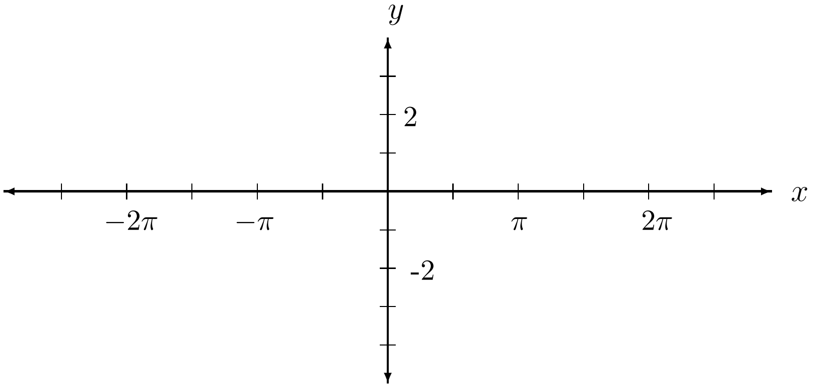
(i) Find the smallest possible value for c

(ii) Determine the angular speed ω of the rotating object.

(44) Fill in the blank.

Function	Amplitude	Period	Phase shift
$y = 2\sin x$			
$y = \cos(2x - 1)$			
$y = -3\sin(3\pi x - \pi)$			

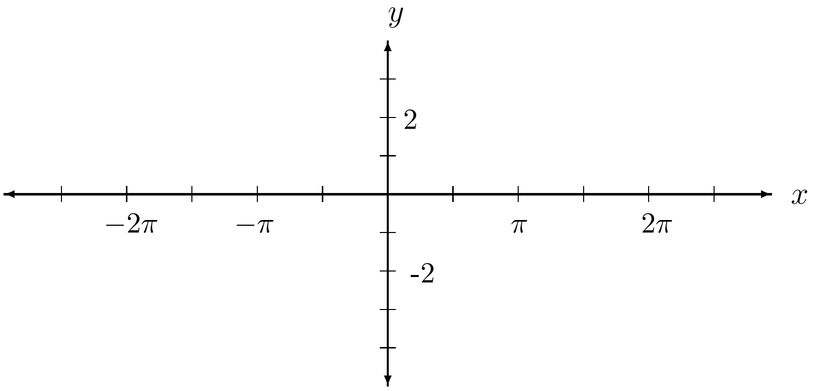
(45) Sketch the curve $y = 2\sin(2x - \pi)$ roughly for $-\pi \le x \le 2\pi$.



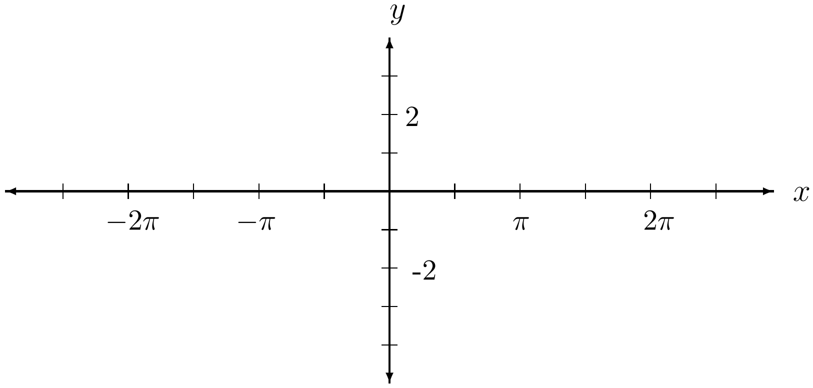
(46) Fill in the blank.

Function	Amplitude	Period	Frequency	Phase shift
$y = -3\sin(3\pi x - 2)$				
$y = \tan(\pi x - \frac{\pi}{2})$				

(47) Graph the trigonometric function $f(x) = 2\sin(\frac{1}{2}x - \frac{\pi}{2})$ for $-2\pi \le x \le 2\pi$.



(48) Graph the trigonometric function $f(x) = -\cos(2x - \frac{\pi}{2})$ for $-2\pi \leq x \leq 2\pi$.



(49) Simplify each trigonometric expression.

a) $\cos(\pi + x) + \sin(\pi + x)$

b) $\dfrac{\tan x + \tan 2x}{1 - \tan x \tan 2x}$

(50) Fill in the blanks to make an identity.

a) $\cos^2 x +$ ____________________ $= 1$.

b) $\sin x \cos y - \cos x \sin y =$ ____________________ .

c) $\sin \dfrac{x}{2} =$ ____________________ .

d) $\cos 2x =$ ____________________ .

e) $1 +$ ____________________ $= \csc^2 x$.

(51) Prove each identity.

a) $\sin(30° - x) + \cos(60° - x) = \cos x$

b) $1 = (\csc x - \cot x)(\csc x + \cot x)$

c) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

(52) Solve each equation for all values in $0 \leq x < 2\pi$.

a) $2 \sec x = 3 - \cos x$

c) $2 \sin x(1 - 2\sin^2 x) = 0$.

(53) Evaluate each determinant. Simplify the result.

a) $\begin{vmatrix} \sin\theta & \cos\theta \\ \csc\theta & \sec\theta \end{vmatrix}$

b) $\begin{vmatrix} 3 & 1 \\ 1 & 1 \end{vmatrix}$

c) $\begin{vmatrix} 1 & 0 & 1 \\ 2 & 2 & 0 \\ 1 & 1 & 1 \end{vmatrix}$

(54) Solve each system of linear equation by determinants (or Cramer's rule.)

$$\begin{cases} 2x - 2y = 2 \\ -2x + y = 3 \end{cases}$$

$$\begin{cases} 3x + y = 5 \\ 2y - 3z = -5 \\ x + 2z = 7 \end{cases}$$

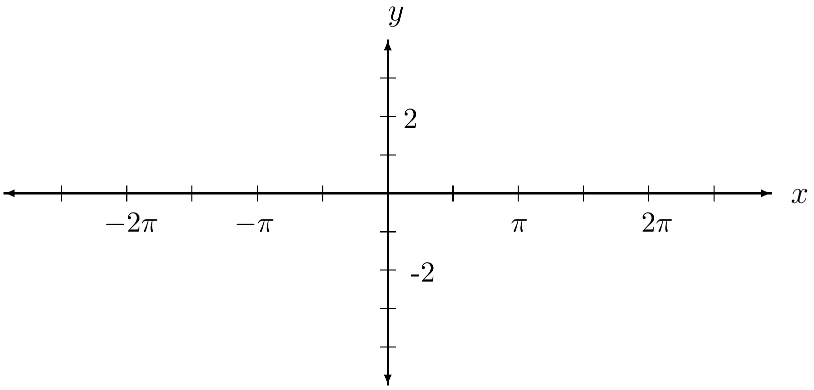
$$\begin{cases} x + y = 2 \\ 3x + 2y = -4 \end{cases}$$

$$\begin{cases} 3x - y + z = 2 \\ 3x + 2y - z = -4 \end{cases}$$

(55) Fill in the blank.

Function	Amplitude	Period	Frequency	Phase shift
$y = 2\sin x$				
$y = \cos(2x - 1)$				
$y = -3\sin(3\pi x - \pi)$				

(56) Sketch the curve $y = -2\sin(2x - \pi)$ roughly for $-\pi \le x \le 2\pi$.

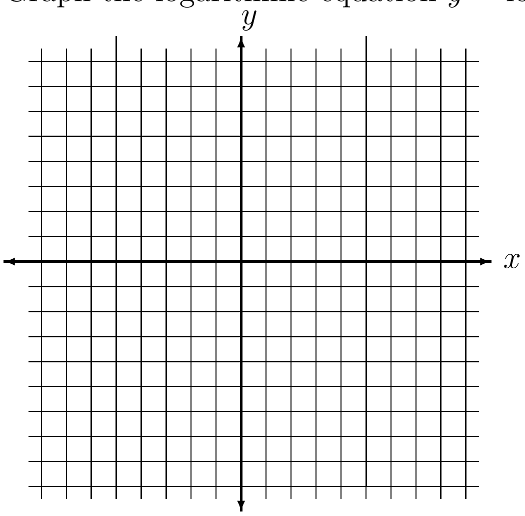


(57) Let $f(t) = 5\sin(2.34t + 3)$ represents the displacement of rotating object. Determine the angular speed ω of the rotating object.

(58) Find the domain of each function

a) $f(x) = \dfrac{11}{x^2 + 3x - 18}$ b) $g(x) = 5^x$ c) $h(x) = \log_5 x$

(59) Graph the logarithmic equation $y = \log_2 x$. Label at lesast three points on the graph.



(60) Let $f(x) = 2x^2 + 3$ and $g(x) = x + 1$. Find

a) $f(3) - g(1)$ b) $\dfrac{f(-1)}{g(-1)}$

(61) Simplify each trigonometric expression.

a) $\cos(\pi + x) + \sin(\pi + x)$ b) $\dfrac{\cos x}{\cot x \sin x}$ c) $\dfrac{\sec x - \csc x}{1 - \cot x}$

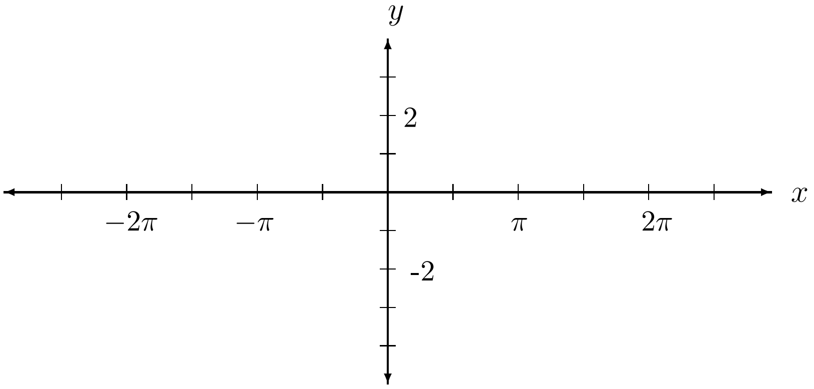
(62) Prove each identity.

a) $\sin(30° + x) + \cos(60° + x) = \cos x$ b) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

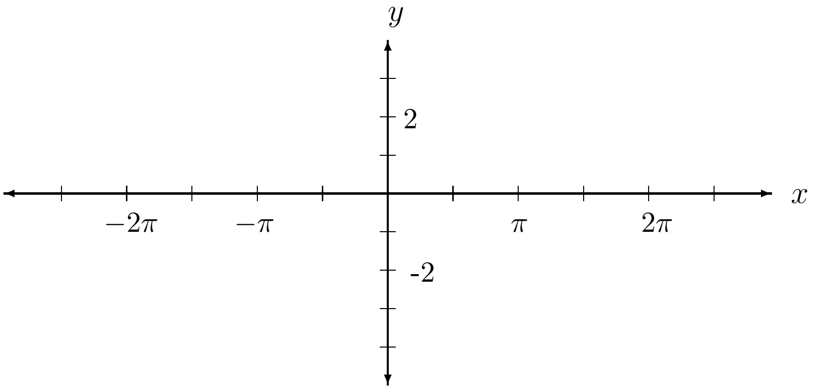
(63) Fill in the blank.

Function	Amplitude	Period	Frequency	Phase shift
$y = -3\sin(3\pi x - 2)$				
$y = \tan(\pi x - \frac{\pi}{2})$				

(64) Graph the trigonometric function $f(x) = 2\sin(\frac{1}{2}x - \frac{\pi}{2})$ for $-2\pi \leq x \leq 2\pi$.



(65) Graph the trigonometric function $f(x) = -\cos(2x - \frac{\pi}{2})$ for $0 \leq x \leq 2\pi$.



(66) Find postive b, c of the function $y = 3cos(bx + c)$ such that the period is 4 and phase shift is $-\frac{2}{\pi}$.

(67) Prove each identity.

a) $\sin(30° - x) + \cos(60° - x) = \cos x$ b) $(\csc x - \cot x)(\csc x + \cot x) = 1$

c) $\cos 2x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$ d) $2\sin^2\left(\dfrac{x}{2}\right) + \cos x = 1$

(68) Find the exact value of $\cos 75°$. No decimal value will be accepted asan answer.

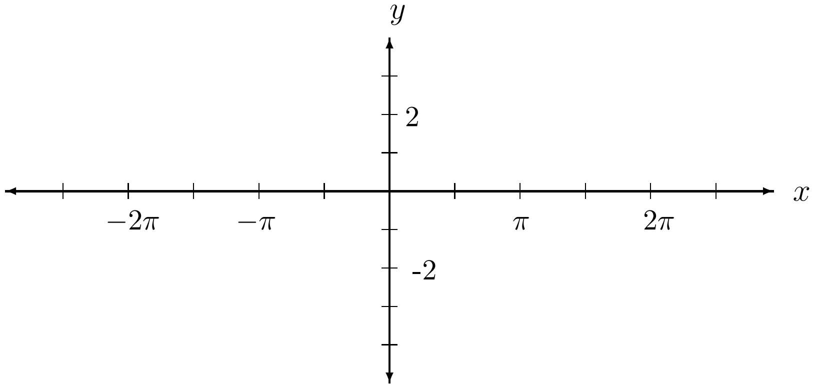
(69) Solve each equation for all values in $0 \leq x < 2\pi$.

a) $2\sec x = 3 - \cos x$ c) $2\sin x(1 - 2\sin^2 x) = 0$.

(70) Fill in the blank.

Function	Amplitude	Period	Frequency	Phase shift
$y = 2\sec x$				
$y = \tan(2x - \pi)$				

(71) Sketch the curve $y = -2\tan(\frac{1}{2}x - \pi)$ roughly for $-\pi \le x \le 2\pi$.



(72) Solve for x.

a) $6^{x+1} = 9$ b) $\log(2x - 5) = 2$

c) $\log_8 8x^2 - \log_8 4x = 1$ d) $\ln(2x - 1) - 2\ln 4 = 3\ln 2$

e) $2^x = 3^{x-1}$

(73) Write as the sum or difference of two or more logarthims.

a) $\log \dfrac{xy^2}{z}$ b) $\log \dfrac{\sqrt{x}}{y}$

(74) Write as a single logarithm.

a) $2\log x - 3\log y$ b) $\log x + \frac{1}{3}\log y - \frac{1}{2}\log z$

(75) Solve for x.

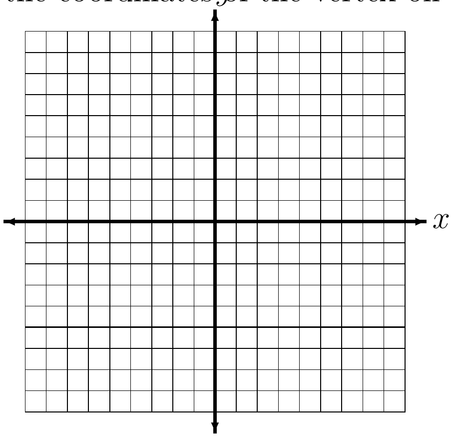
a) $6^{x+1} = 9$ b) $\log(2x - 5) = 2$

c) $\log_8 8x^2 - \log_8 4x = 1$ d) $\ln(2x - 1) - 2\ln 4 = 3\ln 2$

(76) Find the domain of each function.

a) $f(x) = \dfrac{1}{x-1}$ b) $f(x) = \sqrt{x+1}$

(77) Let $f(x) = x^2 + 2x + 3$ be a quadratic function, graph the parabola given by $f(x)$. Indicate the coordinates of the vertex on the parabola.



(78) Let $f(x) = 2^{-x}$, evaluate $f(3)$ and $f(-1)$.

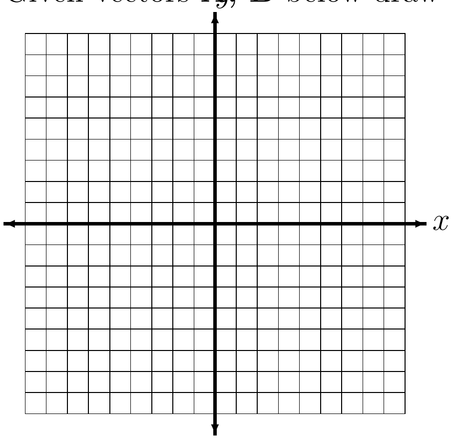
(79) Write the complex number $2 - 3j$ in both polar form and exponential form.

(80) Perform each operation.

a) $j^{13} - j^2$ b) $(7 - 5j) - (-6 + 5j)$ c) $\dfrac{2 - j}{3 + 3j}$

(81) Let $\mathbf{V} = 4.3\angle 16°$ be a complex number. Find the real and imaginary parts of the complex number $\mathbf{V}$.

(82) Given vectors **A**, **B** below draw vector **2A**−**B**



(83) Find the x- and y-components of the vector with magnitude 76.3 mph and angle $\theta = 234°$

(84) A freight bin is lifted in mid-air by two cables. If the force in one cable is 1,200 lbs at 20° from the vertical and the force in other cable is 800 lbs at 40°. What are the resultant force exert on the bin by two cables and direction of lift.

(85) Add two vectors with $A_x = 7, A_y = 9$ and $B_x = -6, B_y = 1$ and give the magnitude and angle of the resultant vector $A + B$.

(86) Add two vectors $A = 10\angle 45°$ and $B = 6\angle 150°$. Write the resulting vector in rectangular form.

a) Divide and express the answer in the rectangular form: $\dfrac{3-4i}{1+2i}$

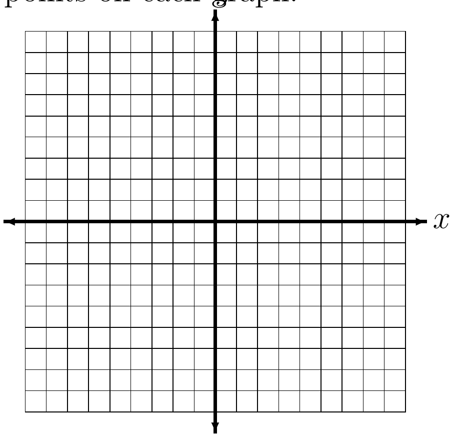
b) Give the polar form of the complex number $(1+i)^4$.

c) Give the rectangular form of the complex number $4\angle 45°$

(87) Given the function $f(t) = \dfrac{5t-3}{t+1}$, find

a) $f(-1)$ and $f(t-1)$. b) domain of $f(t)$.

(88) Graph the both equations $y = 2^x$ and $y = \log_2 x$ on the same set of axes. Label at least two points on each graph.



(89) (i) Solve for x.

a) $\log_4 8 = x$ b) $\log_4 x + \log_4 6 = \log_4 18$

(ii) Write $\frac{1}{2}\log y + \log x - \log 2z$ as a single logarithm.

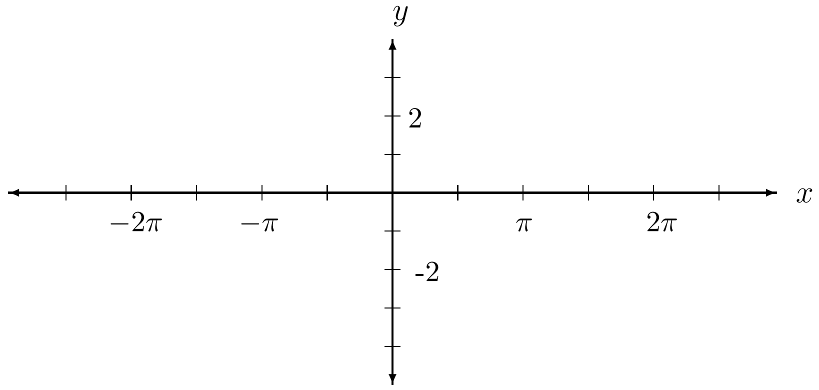
(90) a) Fill in the blanks.

Function	Amplitude	Period	Phase Shift
$y = 2\cos(2x-\pi)$			
$y = -2\tan(2x+3)$			

b) The function of displacement of a point on the wheel of radius 10 is given by $y = 10\sin(\omega t + \alpha)$.

If the period is $\frac{\pi}{10}$ and it has an initial displacement of $\frac{\pi}{3}$, determine the angular velocity ω and α.

(91) Sketch roughly the graph of the function $y = 2\sin(2x - \pi)$ for the interval $[-\pi, 2\pi]$. Label at least three points on the graph.



(92) Prove each identity.

a) $\dfrac{\sin x \cot x + \cos x}{\cot x} = 2\sin x$ b) $\dfrac{\cos 2x}{\cos^2 x} = 1 - \tan^2 x$

(93) Solve each equation for $0 \le x < 2\pi$.

a) $2\cos x - 1 = 0$ b) $-3\cos x + 3 = 2\sin^2 x$

(94) Solve the system of linear equations using the determinants.

$$\begin{cases} x + 2y = 0 \\ -x + y + 2z = 3 \\ -3y + z = 7 \end{cases}$$

(95) Evaluate

a): $\sqrt{-81} =$

b): $\sqrt{(-2)(-8)} =$

c): $\sqrt{-2}\sqrt{-8} =$

d): $i^{403} =$

e): $(-1)^{1001} =$

(96) **a):** $8 - 15 - 5 \cdot 4 =$

b): $-5 + (4^2 - (2)(7))^2 =$

(97) **a):** $3i - \sqrt{-100} =$

b): $i^2 - i^6 =$

(98) **a):** $(3 - 4i) - (8 + 6i) =$

b): $(3 - 4i)(8 + 6i) =$

(99) **a):** Perform the same multiplication $(3-4i)(8+6i)$ in polar form (i.e., by first converting the numbers $3 - 4i$ and $8 + 6i$ into polar form and then using polar multiplication).

b): Convert your result back to rectangular form and compare with the result of the previous problem (i.e., Problem **3b)**).

(100) Round your answer to 3 digits after the point

a): one radian in degrees:

b): one degree in radians:

(101) Write $2i$ in

a): polar form:

b): exponential form:

(102) **a):** Write $\frac{4(\cos 120° + i \sin 120°)}{2(\cos 30° + i \sin 30°)}$ in polar form (i.e., divide!).

b): Write the result (i.e. the same fraction) in rectangular form (you may just use a picture for that!).

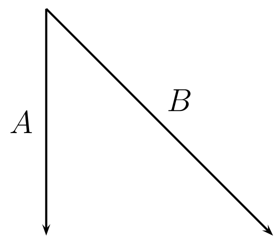
(103) Find all cubic roots of 8,

a): first in polar form:

b): then in rectangular form:

(104) **a):** Add the following two vectors.

b): Perform the subtraction $\mathbf{B} - \mathbf{A}$.



(105) A ski tow is moving skiers vertically upward at $24\, m/min$ and horizontally at $44\, m/min$. What is the resultant velocity of the tow? (Think: is it a vector or a number?)

(106) **a):** (**Without calculator: show your calculation!**) Find the base b such that $b^{1/3} = 3$.

b): (**Without calculator: show your calculation!**) Find $f(-2/3)$ for $f(x) = 8^x$.

(107) **a):** $8 - 15 - 5 \cdot 4 =$

b): $-5 + (4^2 - (2)(7))^2 =$

(108) To save time simply write the answer without justification (**no calculator!**).

a): $\log_3 1 =$

b): $\log_3 3 =$

c): $\log_\pi \pi =$

d): $\log_2 8 =$

e): $\log_2 \frac{1}{4} =$

(109) **a):** Find b if $\log_b 10000 = 4$.

b): Find b if $\log_b 2 = -\frac{1}{2}$.

(110) **a):** Find the exact value: $\log_5 \sqrt[3]{5} =$

b): Solve $\log_4 y = \log_4 x - \log_4 3$ for y in terms of x.

(111) Find $\log_2 86$ (using the calculator).

(112) Find the resultant of the following three vectors with their magnitudes given first followed by their direction as an angle in standard position.

$\vec{A} : 422, 0°$; $\vec{B} : 210, 110°$; $\vec{C} : 405, 235°$.

(113) A ship sails $32.50mi$ due east and then turns $41.25°$ north of east. After sailing another $16.18mi$, where is it with reference to the starting point?

(114) Express the side length of a square as a function of its area. (Explain what the variables used stand for.)

(115) Consider the function f given by $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ 1 & \text{if } x \geq -1 \end{cases}$

a): Find the values of f at $x = -3, -2, -1, 0, 1, 2, 3$.

b): What is the domain of f?

(116) **a):** Graph the function f from the previous problem.

b): What is the range of f?

(117) Solve $3^{x-2} = 5$ for x.

(118) **a):** $8 - 15 - 5 \cdot 4 =$

b): $-5 + (4^2 - (2)(7))^2 =$

(119) Using Cramer's Rule, solve the system below for x and y.

$$\begin{array}{rcrcl} \text{x} & + & \text{y} & = & 36 \\ 0.25\text{x} & + & 0.375\text{y} & = & 11.2 \end{array}$$

(120) Compute

$$\begin{vmatrix} 1 & 5 & 4 \\ -2 & 3 & -1 \\ 2 & -1 & 5 \end{vmatrix}$$

(121) Prove the identity $1 + \tan^2 x = \sec^2 x$.

(122) Solve $2\cos x + 1 = 0$ for x.

(123) Consider an angle x in the second quadrant. If $\sin x = -\frac{1}{2}$, find $\sin \frac{x}{2}$.

(124) Given that $\sin\alpha = \frac{5}{13}$ (α in the first quadrant) and $\sin\beta = -\frac{3}{5}$ (β in the third quadrant), find $\cos(\alpha+\beta)$.

(125) Given the function $y = -2\sin(3x - \pi)$,

a): what is its period?

b): what is its amplitude?

c): what is its phase shift?

(126) Graph the function $y = \tan x$.

(127) Graph the function $\cos(2x)$.

(128) **a):** $\sqrt{-81} =$

b): $i^{41} =$

c): $\log_3 1 =$

d): $\log_2 8 =$

e): $\log 1000 =$

(129) Add two vectors $A = 10\angle 45°$ and $B = 6\angle 150°$. Round all results to 2 digits after the point.

(130) A river flows due South at a rate of 3 mph. A rower who travels 4 mph in stillwater, heads due East, exactly across the current. Find the velocity of the boat to the nearest tenth. [Think: are you looking for a number or for a vector?]

(131) Divide and express the answer in rectangular form: $\dfrac{3-4i}{1+2i}$

(132) Give the polar form of the complex number $1 + i$.

(133) Using the previous answer and DeMoivre's Theorem, find the polar form of the complex number $(1+i)^4$.

(134) Find the rectangular form of the complex number $2\angle 30°$.

(135) Find the exponential form of the complex number $2\angle 30°$.

[Hint: the exponential form uses imaginary powers of the number e with angles in radians.]

(136) Given the function $f(t) = \dfrac{5t-3}{t+1}$, find

a) $f(-1) =$

b) $f(1) =$

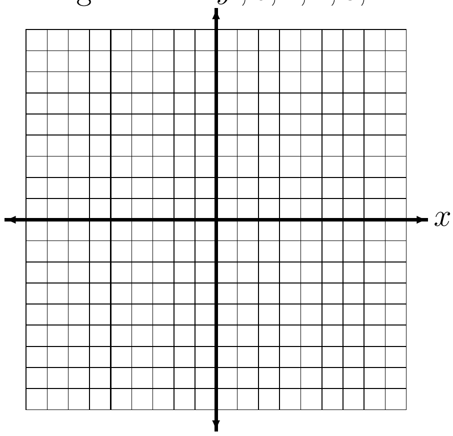
c) the domain of $f(t)$

(137) Solve for x: $\log_4 8 = x$

(138) Write $\frac{1}{2}\log y + \log x - \log 2z$ as a single logarithm.

(139) Solve for x and round to 4 digits after the point: $3^{(x-5)} = 1000$

(140) Graph the equations $y = 2^x$ and $y = \log_2 x$ on the same set of axes. [Hint: plot the points on the graph of the exponential function corresponding to $x = -1, 0, 1, 2, 3$; for the logarithmic

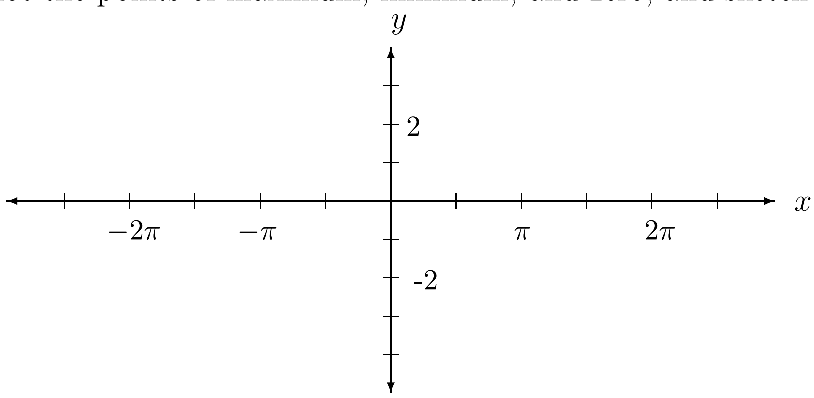


function, do the same with $x = \frac{1}{2}, 1, 2, 4, 8$.]

(141) Fill in the blanks.

Function	Amplitude	Period	Phase Shift
$y = 3\sin(2x - \pi)$			

(142) Consider the above function $y = 3\sin(2x - \pi)$ on the interval $[0, 2\pi]$ (i.e., from 0 to 2π). Plot the points of maximum, minimum, and zero, and sketch the graph.



(143) Prove the identity $\dfrac{\sin x \cot x + \cos x}{\cot x} = 2\sin x$

(144) Solve the equation for x with $0 \le x < 2\pi$: $2\cos x - 1 = 0$

(145) Write down the sum formula for cosine: $\cos(\alpha + \beta) =$

(146) Find the exact value of $\cos 75°$. No decimal value will be accepted as an answer (i.e., do not use the calculator, though you certainly may check your answer using it).

(147) Solve the system of linear equations using Cramer's Rule. $\begin{cases} x + 2y = 0 \\ -x + y + 2z = 3 \\ -3y + z = 7 \end{cases}$

(148) **(Extra Credit)**

Find all cubic roots of $8i$ (in polar form, to make it simple). [How many are there?] a) Find the rectangular form of the complex number $(1+i)^4$, whose polar form you found in

Problem 133. b) Compute the 4th power of $1 + i$ directly (i.e., algebraically) and compare your result with what you got in a).

Find the rectangular forms of the cubic roots of $8i$ that you found, in polar form, in Problem 148.

(149) Graph the function $y = \tan x$.

(150) Graph the function $\cos(2x)$.