WORKBOOK. MATH 12. INTRODUCTION TO MATHEMATICAL THOUGHT

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE

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Contents

1. Introduction, Notation and Description, Subsets

(1) What is a set? Give three examples of finite sets.

(2) Give three examples of infinite sets. You will need to use ellipses (*singular form*: ellipsis).

(3) Write the following sets:
  (a) The set of days in a week
    - in **roster form**
    - in **set-builder notation**
  (b) The set of months in a year
    - in **roster form**
    - in **set-builder notation**
  (c) The set of natural numbers
    - in **roster form**
    - in **set-builder notation**
  (d) The set of whole numbers
    - in **roster form**
    - in **set-builder notation**
  (e) The set of prime factors of 120.
    - in **roster form**
    - in **set-builder notation**

(4) Explain what the following notations stand for. Present an example for each of the notations.
  - $\{\quad\}$
  - $\in$
  - $\notin$
  - $\cdots$
  - $\subset$

- $\subseteq$
- $\not\subseteq$
- U
- A'
- $\cap$ (Present three examples)
- $\cup$ (Present three examples)
- $n(A)$
- (a, b)
- $A \times B$
- $a \mid b$
- $n!$ for a whole number n

(5) What is a well-defined set? Give an example of a set which is not well-defined.

(6) What is an empty set. How do you denote an empty set?

(7) Is $\{\phi\}$ an empty set? Explain.

(8) When is a set A said to be a subset of set B? Explain using an example.

(9) When is a set A said to be a **proper** subset of set B? Explain using an example.

(10) Give an example of two sets A and B such that $A \not\subset B$.

(11) Is $A \not\subset B$ same as $A \nsubseteq B$? Explain.

(12) Which set is a subset of every set?

(13) List all the subsets of

- ϕ
- $\{a\}$
- $\{\phi\}$
- $\{a, b\}$
- $\{a, b, c\}$

(14) A set of cardinality 0 has ____________ subsets.
(15) A set of cardinality 1 has ____________ subsets.
(16) A set of cardinality 2 has ____________ subsets.
(17) A set of cardinality 3 has ____________ subsets.
(18) A set of cardinality 4 has ____________ subsets.
(19) A set of cardinality n has ____________ subsets.

(20) Explain using an example the notion of the complement of a set A in a universal set U.

(21) Let $U = \{1, 2, 3, 4, 5, 6, 7\}$ be the universal set. Find the complement of

- $A = \{1, 2\}$
- $B = \{2, 3, 4\}$
- $C = \phi$

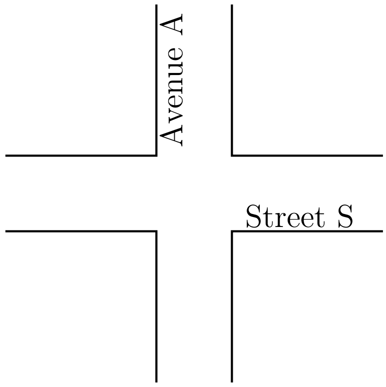
(22) State true or false:

- $\{\} \subseteq \{\}$
- $\{\} \subset \{\}$
- $\{2, 3\} \subset \{1, 2, 3\}$
- $\{2\} \subset \{1, 2, 3\}$
- $2 \subset \{1, 2, 3\}$
- $\{2\} \in \{1, 2, 3\}$
- $2 \in \{1, 2, 3\}$
- $\{A, B\} \subset \{a, b, c\}$
- $\{a, b\} \subseteq \{a, b, c\}$
- $\{a, b\} = \{a, b, c\}$
- $\{1, 2, 3\}$ is equivalent to $\{a, b, c\}$
- $\{\} = \phi$
- $\{\phi\} = \phi$

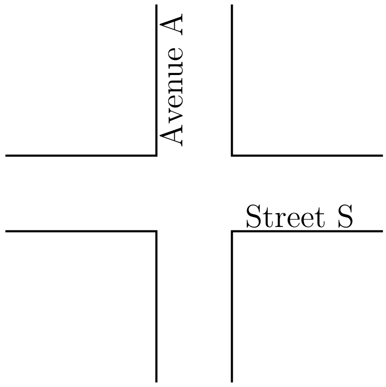
(23) There are two one-dollar bills, two five-dollar bills, and one ten-dollar bills on a table. You are allowed to select three bills. List all possible selections. What is the least amount of money you can select? What is the greatest amount of money you can select?

2. Set operations and Venn diagrams

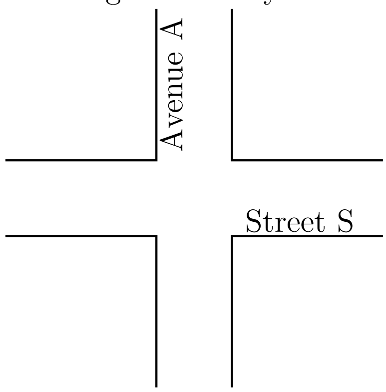
(1) You are standing on Avenue A and on Street S. Shade the region where you could be. What is that region called?



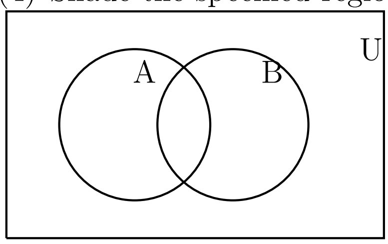
(2) You are standing on Avenue A or on Street S, but not on both Avenue A and Street S. Shade the region where you could be.



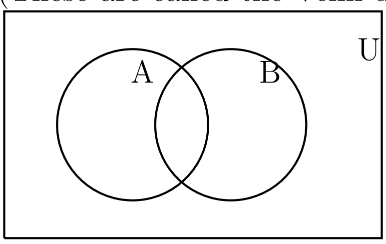
(3) You are standing on Avenue A or on Street S. Shade the region where you could be.



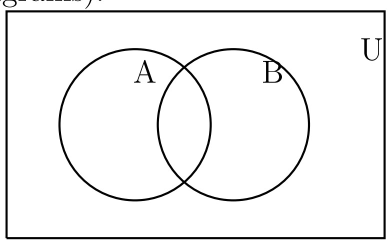
(4) Shade the specified region (These are called the Venn diagrams):



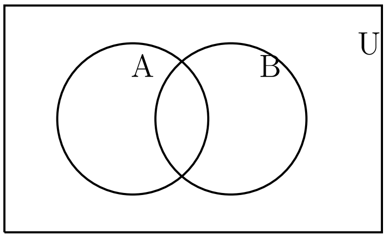
A and B
A *intersection* B
$A \cap B$



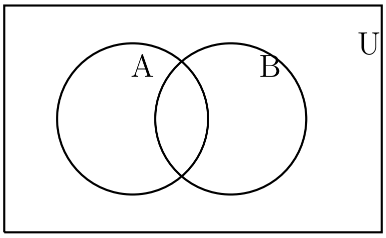
A or B but not both



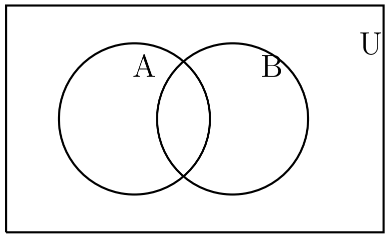
A or B
A *union* B
$A \cup B$



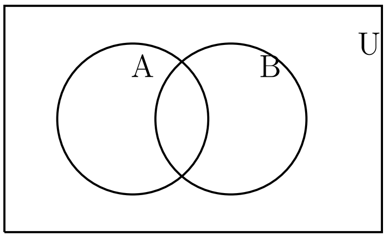
not A
A *complement*
$A' = A^c = \overline{A}$



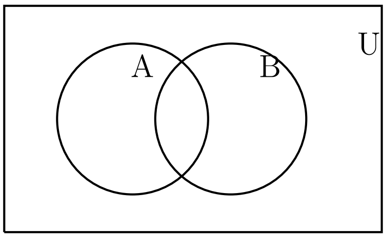
not B
B *complement*
$B' = B^c = \overline{B}$



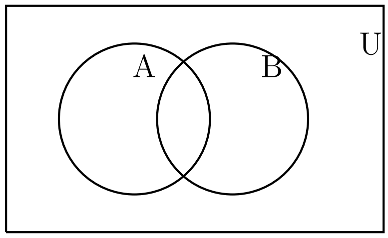
$A \cup A$



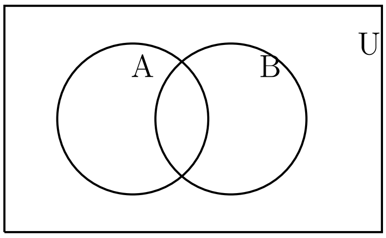
$(A \cup B)'$



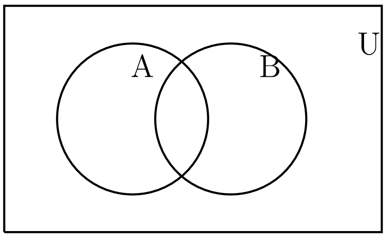
$(A \cap B)'$



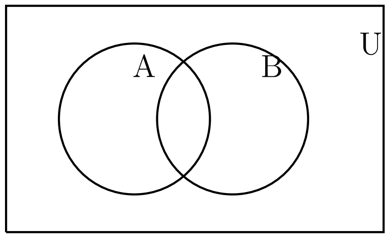
$A \cap A$



$A' \cup B'$



$A' \cap B'$



$A \cup \phi$. What is $A \cap \phi$?

(5) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{3, 4, 5, 6\}$, $B = \{5, 6, 7, 8\}$, and $C = \{1, 2, 5, 7\}$. Find

- $A \cup B$
- $B \cup A$
- $A \cup A$
- $A \cap B$
- $B \cap A$
- $B \cap B$
- $C \cap \phi$
- $B \cup \phi$
- $A \setminus C$
- $A \setminus B$
- $A \setminus (B \cup C)$
- $(A \setminus B) \cup (A \setminus C)$
- $A \setminus (B \cap C)$
- $(A \setminus B) \cap (A \setminus C)$
- $A \cup (B \cup C)$
- $(A \cup B) \cup C$
- $A \cap (B \cap C)$
- $(A \cap B) \cap C$

- $A \cup (B \cap C)$
- $(A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C)$
- $(A \cap B) \cup (A \cap C)$

(6) State the Set laws:

Empty set laws:

Idempotency laws:

Commutative laws:

Associative laws:

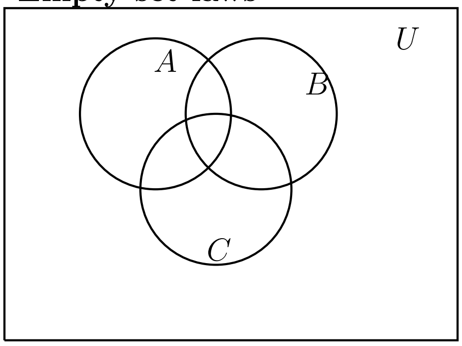
Distributive laws:

Absorption laws:

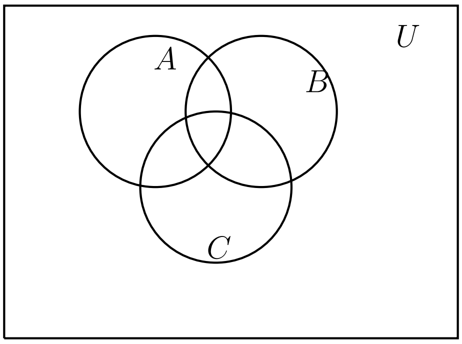
DeMorgan's laws:

(7) Explain the Set laws using the Venn diagrams.

Empty set laws

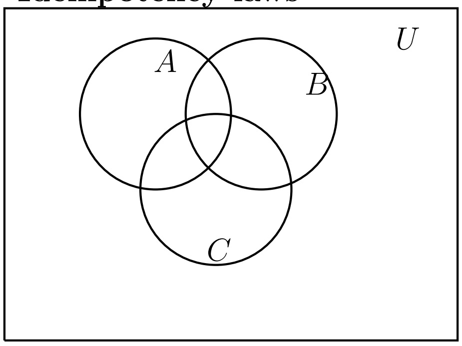


$A \cap \phi$

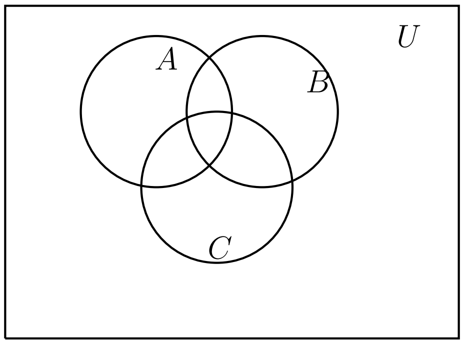


$A \cup \phi$

Idempotency laws

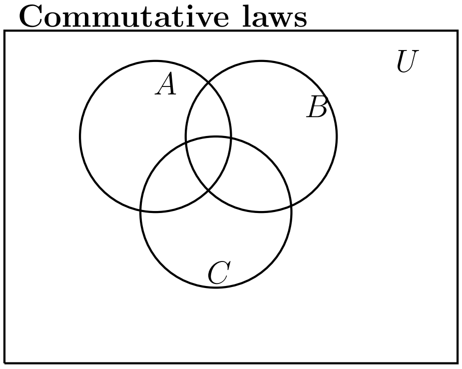


$A \cap A$

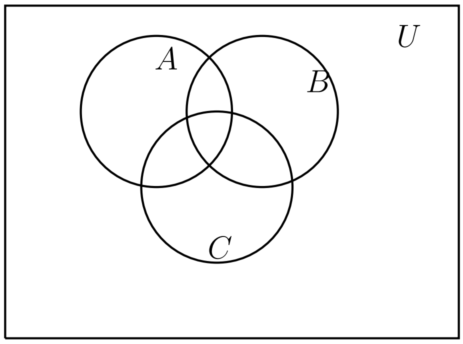


$A \cup A$

Commutative laws

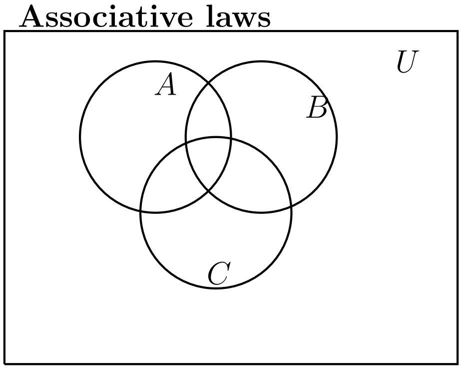


$A \cap B$
$B \cap A$

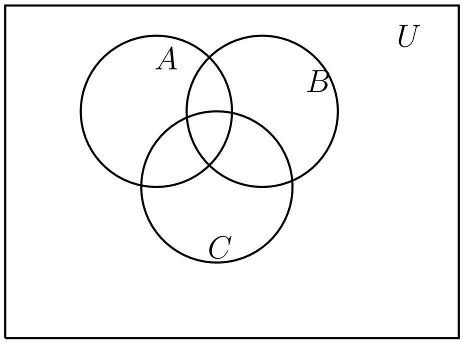


$A \cup B$
$B \cup A$

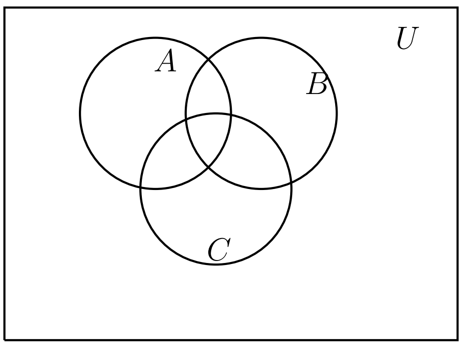
Associative laws



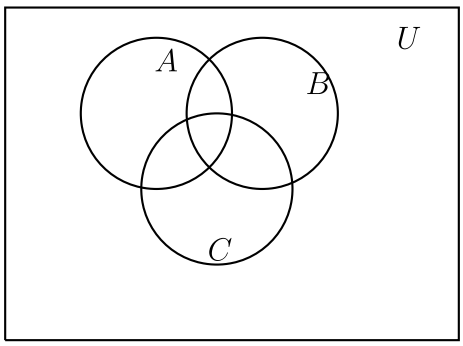
$(B \cap C)$



$A \cap (B \cap C)$

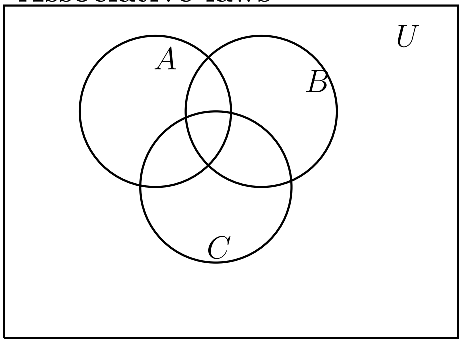


$(A \cap B)$

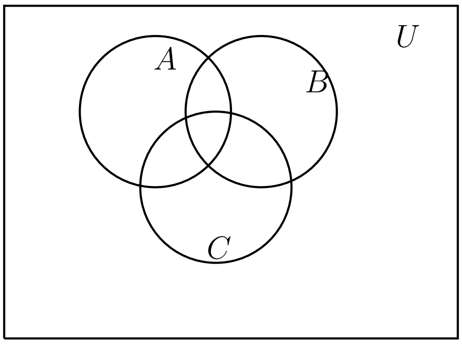


$(A \cap B) \cap C$

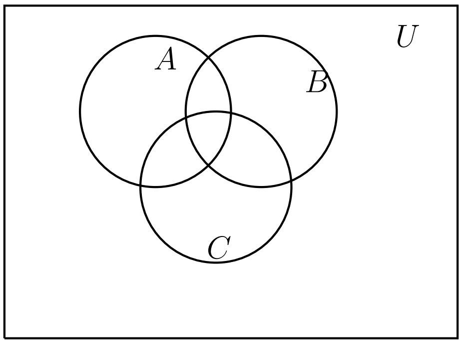
Associative laws



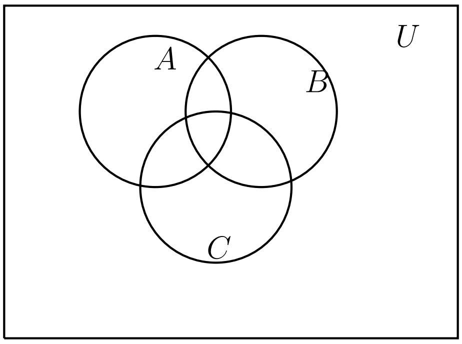
$(B \cup C)$



$A \cup (B \cup C)$

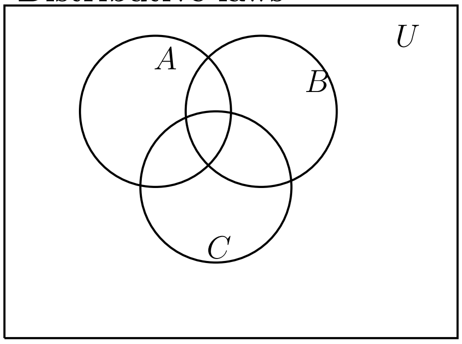


$(A \cup B)$

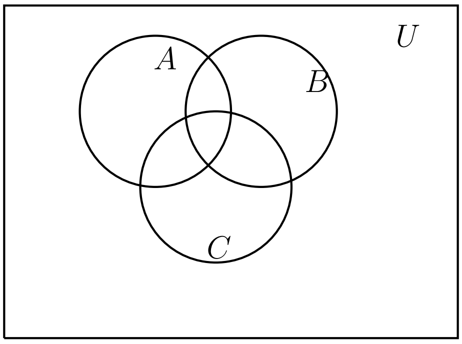


$(A \cup B) \cup C$

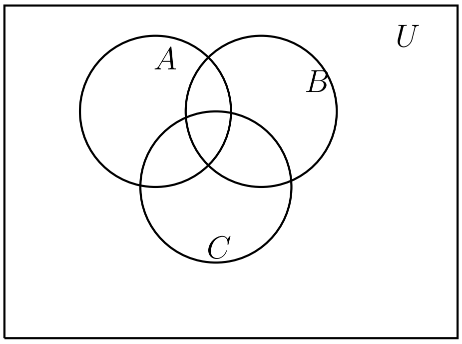
Distributive laws



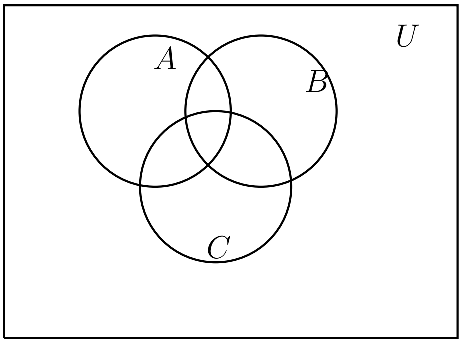
$(B \cap C)$



$A \cup (B \cap C)$

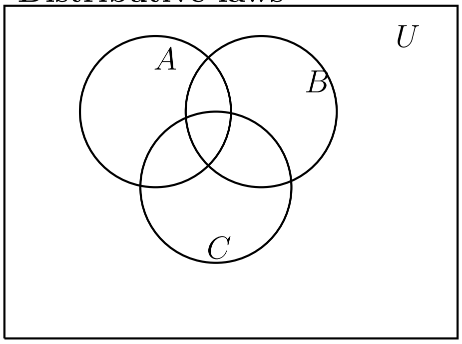


$(A \cup B)$

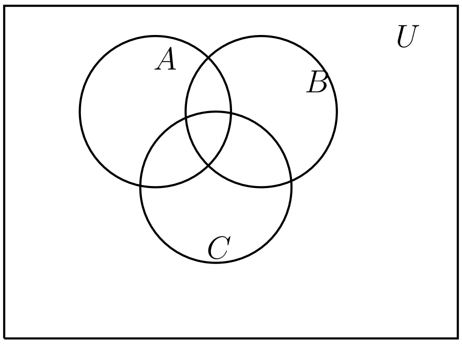


$(A \cup C)$

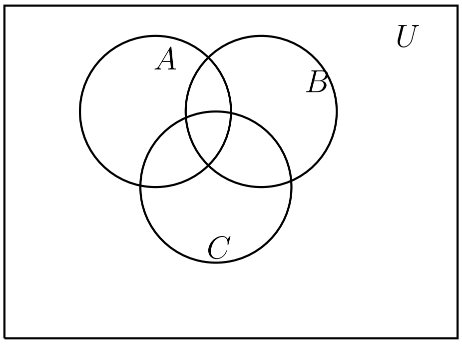
Distributive laws



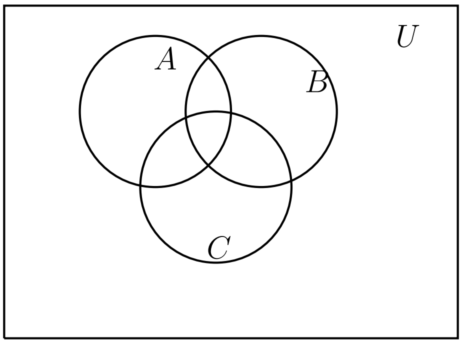
$(B \cup C)$



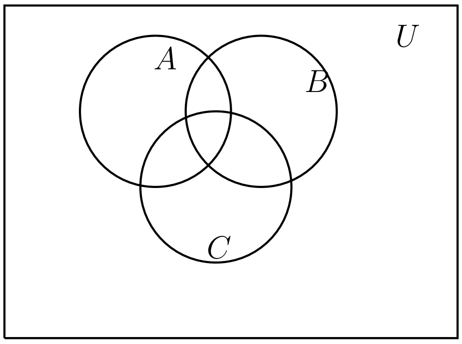
$A \cap (B \cup C)$



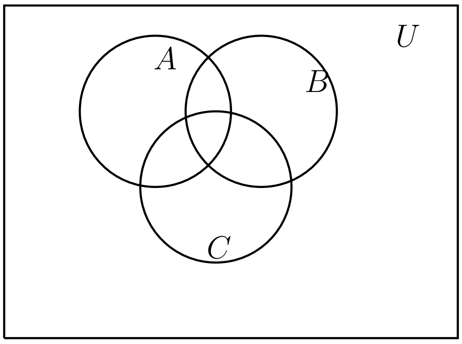
$(A \cap B)$



$(A \cap C)$

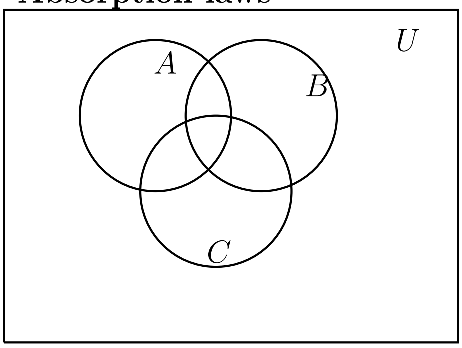


$(A \cap B) \cup (A \cap C)$

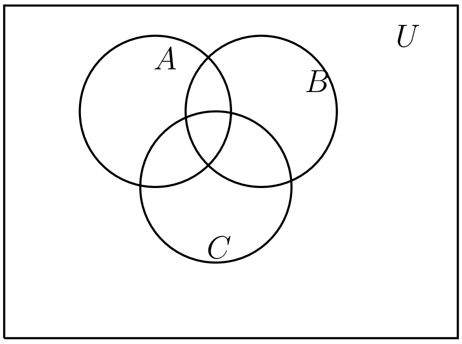


$(A \cup B) \cap (A \cup C)$

Absorption laws

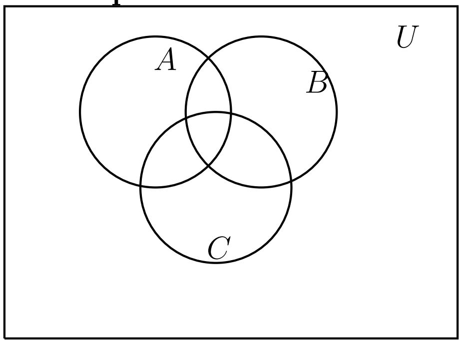


$(A \cup B)$

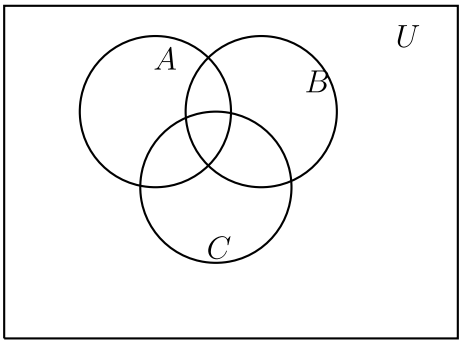


$A \cap (A \cup B)$

Absorption laws

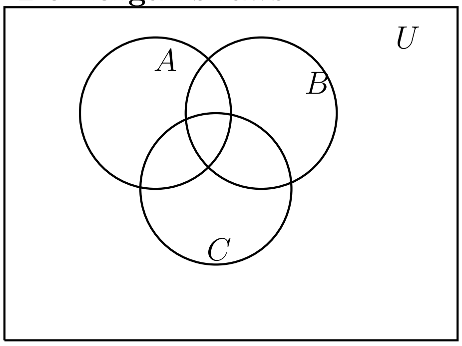


$(A \cap B)$

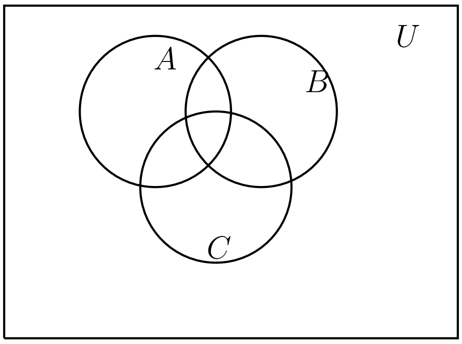


$A \cup (A \cap B)$

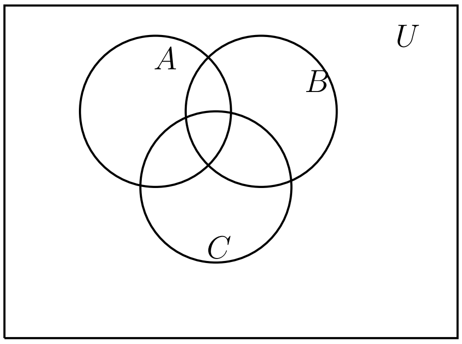
DeMorgan's laws



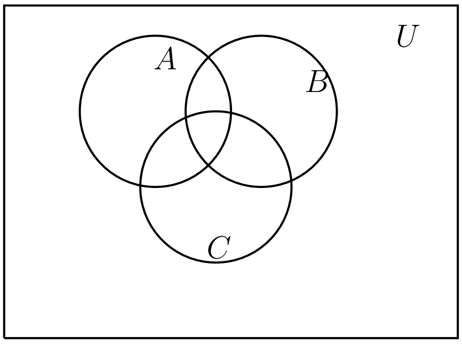
$(B \cap C)$



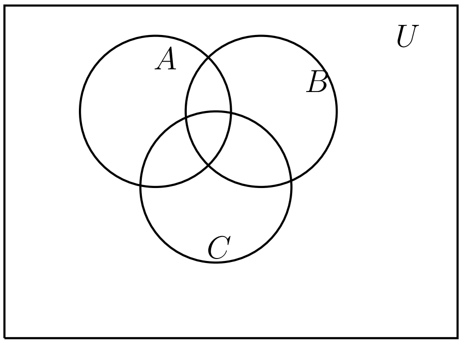
$A \setminus (B \cap C)$



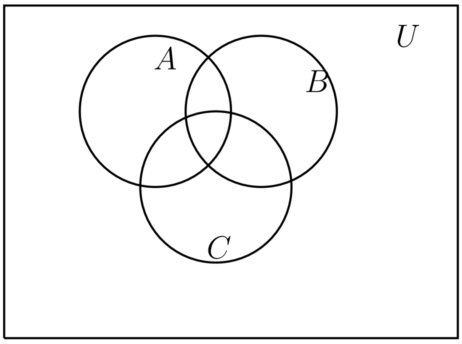
$A \setminus B$



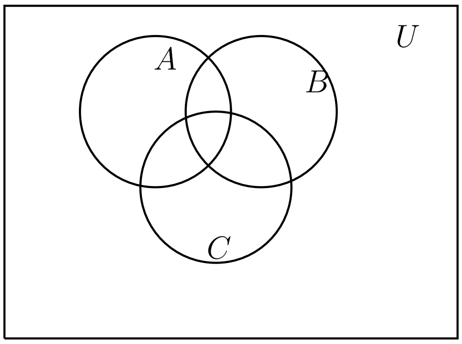
$A \setminus C$



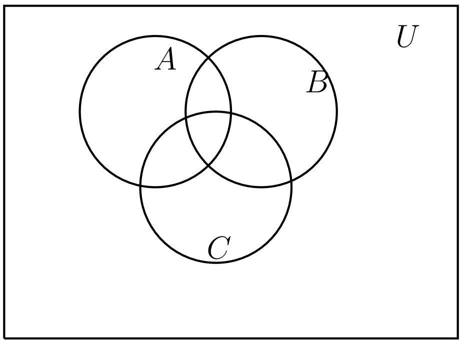
$(B \cup C)$



$A \setminus (B \cup C)$



$(A \setminus B) \cup (A \setminus C)$



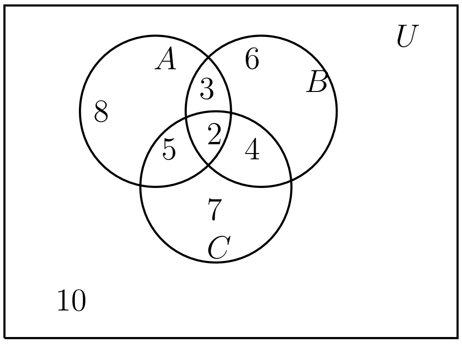
$(A \setminus B) \cap (A \setminus C)$

(8) What is the cardinality of a set? Explain using two examples.

(9) What is the cardinality of

- $\{\ \}$
- $\{0\}$
- $\{\phi\}$
- $\{\{\ \}\}$
- $\{1, 2, 3\}$
- $\{a, b, A\}$
- $\{1, 2, 3, \cdots\}$

(10) Use the Venn diagram to find



$n(A)$
$n(A \cap B)$
$n(A \cup B)$
$n(A) + n(B)$
$n(A) + n(B) - n(A \cap B)$
$n(A' \cap B' \cap C)$
$n(A' \cap B \cap C)$
$n(A \cup B \cup C)$
$n((A \cup B \cup C)')$
$n(U)$

(11) What is a one-to-one correspondence between two sets. Present a one-to-one correspondence between two finite sets of the same cardinality. Present a one-to-one between two countably infinite sets.

(12) Describe all the one-to-one correspondences between

- $\{a, b\}$ and $\{1, 2\}$
- $\{a, b, c\}$ and $\{1, 2, 3\}$
- $\{a, b, c, d\}$ and $\{1, 2, 3, 4\}$
- $\{a\}$ and $\{1\}$

(13) Suppose sets A and B are such that $n(A) = n(B) = n$. How many one-to-one correspondeces can be established between A and B? Explain

(14) Can you establish a one-to-one correspondence between the sets $\{1, 2, 3, \cdots\}$ and $\{3, 6, 9, \cdots\}$? Explain.

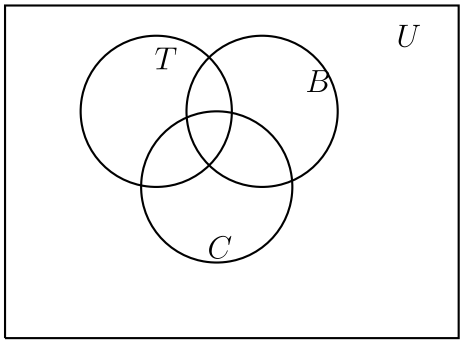
(15) Can you establish a one-to-one correspondence between the sets $\{1, 2, 3, 4\}$ and $\{a, b, c\}$? Explain.

3. An application of Sets and Venn diagrams

The following questions require drawing of Venn diagrams.

(1) In a sample of 200 students, 65 students preferred drinking tea, 80 students preferred drinking coffee, 57 students preferred drinking bottled water, 25 students preferred both tea and coffee, 27 students preferred both coffee and bottled water, 22 students preferred both tea and bottled water, and 12 students liked all the three drinks.

- How many students prefer some other drink?
- How many students prefer tea but not coffee?
- How many students like tea or coffee?
- How many students prefer tea or coffee but not bottled water?



(2) Sixty children were asked to name the colours they liked. Twenty-six children named green, twenty-two named red, twenty-one named blue, eight named green and red, seven named red and blue, nine named green and blue, while three named all the three colours.
- How many children named some other colour?
- How many children named green but not red or blue?
- How many children named green or blue but not red?
- How many children named green, red or blue, but not all the three.

The following are questions from the textbook (section 2.6)

(3) In a recent survey of 300 people regarding television programming, the following information was gathered : 160 people watched ABC, 150 people watched CBS, and 150 people watched NBC, 90 people watched both ABC and CBS, 70 people watched CBS and NBC, and 100 people watched ABC and NBC. Forty people watched all three networks.

- How many people watched ABC or NBC?
- How many people watched only one of the networks?
- How many people did not watch any of the networks?
- How many people did not watch NBC?

(4) Ms. Commission, an investment advisor, analyzed the investments of her clients. She noted that 13 of her clients had invested in stocks, bonds, and mutual funds. Of the 53 clients who invested in mutual funds, 27 also had invested in bonds, while 28 had also invested in stocks. Also, of the 55 clients who had invested in bonds, 29 had invested in stocks. Ms. Commission also noted that she had 54 clients who had invested in stocks, and finally, she had 9 clients who had invested only in other areas.

- How many clients does Ms. Commission have altogether?
- How many clients have invested in stocks or bonds?
- How many clients have not invested in stocks?
- How many of the stock investors have not invested in mutual funds?

4. Statements, Symbols, and Truth-tables

(1) Give two examples of sentences.

(2) What is a statement? Give two examples of statements. In addition, give two examples of sentences which are not statements.

(3) What is a paradox? Give two examples.

(4) What is a simple statement? Give two examples.

(5) What is a compound or complex statement? Give two examples.

(6) What is a negation? Give two examples.

(7) What is a conjunction? Give two examples.

(8) What is a disjunction? Give two examples.

(9) Explain using examples the difference between an inclusive and an exclusive "or".

(10) What is a conditional? Give two examples. In each of your examples, clearly point out the antecedent and the consequent.

(11) What is a biconditional? Give two examples.

(12) Copy down the table of symbols from page 125.

(13) Let $P =$ "Sam is using an umbrella," $Q =$ "It is raining," and $R =$ "Today is a Sunday." Write each of the following statements in words:

- $\sim P$
- $\sim Q$
- $\sim R$
- $P \wedge Q$
- $P \wedge R$
- $Q \wedge R$
- $P \vee Q$
- $P \vee R$

- $Q \vee R$
- $P \underline{\vee} Q$
- $P \underline{\vee} R$
- $Q \underline{\vee} R$
- $\sim P \vee Q$
- $\sim P \vee R$
- $\sim Q \vee R$
- $P \vee \sim Q$
- $P \vee \sim R$
- $Q \vee \sim R$
- $\sim (P \vee Q)$
- $\sim (P \vee R)$
- $\sim (Q \vee R)$
- $\sim (P \wedge Q)$
- $\sim (P \wedge R)$
- $\sim (Q \wedge R)$
- $\sim P \vee \sim Q$
- $\sim P \vee \sim R$
- $\sim Q \vee \sim R$
- $\sim P \wedge \sim Q$
- $\sim P \wedge \sim R$
- $\sim Q \wedge \sim R$
- $P \rightarrow Q$

- $Q \to R$
- $R \to P$
- $P \leftrightarrow Q$
- $Q \leftrightarrow R$
- $R \leftrightarrow P$

(14) Complete the following truth table for conjunction using the following example as a guide: $P =$ "I am standing on Avenue A" and $Q =$ "I am standing on Street S."

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

(15) Complete the following truth table for conjunction using the following example as a guide: $P =$ "x is a number less than 20" and $Q =$ "x is a number greater than 3."

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

(16) Truth table is independent of the fact whether the statements make sense or not. Complete the following truth table for conjunction using the following example as a guide: $P =$ "Blah blah is nonsense" and $Q =$ "Blah blah is garbage."

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

(17) Complete the following truth table for disjunction using the following example as a guide: $P =$ "I am standing on Avenue A" and $Q =$ "I am standing on Street S."

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

(18) Truth table is independent of the fact whether the statements make sense or not. Complete the following truth table for disjunction using the following example as a guide: $P =$ "Blah blah is nonsense" and $Q =$ "Blah blah is garbage."

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

(19) Complete the following truth table for exclusive disjunction using the following example as a guide: $P =$ "I am standing on Avenue A" and $Q =$ "I am standing on Street S."

P	Q	$P \underline{\vee} Q$
T	T	
T	F	
F	T	
F	F	

(20) Truth table is independent of the fact whether the statements make sense or not. Complete the following truth table for exclusive disjunction using the following example as a guide: $P =$ "Blah blah is nonsense" and $Q =$ "Blah blah is garbage."

P	Q	$P \underline{\vee} Q$
T	T	
T	F	
F	T	
F	F	

(21) Construct truth tables for

- $\sim (P \vee Q)$
- $\sim (P \wedge Q)$
- $\sim P \vee Q$
- $P \vee \sim Q$
- $\sim P \wedge Q$
- $P \wedge \sim Q$
- $\sim (P \underline{\vee} Q)$
- $\sim P \underline{\vee} Q$
- $P \underline{\vee} \sim Q$

5. Conditional and Biconditional, DeMorgan's law and Equivalent statements

(1) Construct the truth table for $P \to Q$ using the example: $P =$ "It is raining," and $Q =$ "I am carrying an umbrella." That is, $P \to Q =$ "If it is raining, then I am carrying an umbrella."

P	Q	$P \to Q$
T	T	
T	F	
F	T	
F	F	

(2) Truth table is independent of the fact whether the statements make sense or not. Complete the following truth table for conditional using the following example as a guide: $P =$ "Blah blah is nonsense" and $Q =$ "Blah blah is garbage."
That is, $P \to Q =$ ____.

P	Q	$P \to Q$
T	T	
T	F	
F	T	
F	F	

(3) Recall the truth tables for $P \vee Q$ and $P \wedge Q$:

P	Q	$P \vee Q$
T	T	
T	F	
F	T	
F	F	

P	Q	$P \wedge Q$
T	T	
T	F	
F	T	
F	F	

(4) Now construct the truth table for $P \leftrightarrow Q$. Recall that $P \leftrightarrow Q = (P \to Q) \wedge (Q \to P)$.

P	Q	$P \to Q$	$Q \to P$	$P \leftrightarrow Q$
T	T			
T	F			
F	T			
F	F			

(5) What is a tautology? Explain with an example the statement $P \vee \sim P$.

(6) What is a contradiction? Explain with an example the statement $P \vee \sim P$.

(7) When are two statements said to be logically equivalent? Study the equivalence of the statements $(\sim P \vee Q)$ and $(P \to Q)$.

(8) Construct the truth tables for

- $(P \vee Q) \to (P \wedge Q)$

- $(\sim P \wedge Q) \vee P \to (P \wedge Q)$

- $P \leftrightarrow \sim Q$

- $(\sim P \wedge Q) \leftrightarrow \sim R$

(9) Are the following statements logically equivalent?

- $(\sim P \wedge Q)$ $\qquad$ $(P \vee \sim Q)$

- $(P \wedge \sim Q)$ $\qquad$ $(\sim P \wedge \sim Q)$

- $(P \vee \sim Q)$ $\qquad$ $(Q \rightarrow P)$

(10) Recall that $(P \rightarrow Q)$ and $(\sim P \vee Q)$ are logically equivalent. Write the statements $(P \rightarrow Q)$ and $(\sim P \vee Q)$ in words when

- $P =$ "The sky is blue," and $Q =$ "The water is brackish."

- $P =$ "This is a table," and $Q =$ "This has two legs."

(11) State and prove the laws of equivalent statements listed on page 160.

- DeMorgan's laws:

- Implication:

- Contraposition:

- Biconditional (this is just the definition):

- Association:

- Distribution:

- Idempotent:

(12) Use DeMorgan's laws to create equivalent statements:

- $\sim (P \wedge Q)$
- $\sim (Q \vee \sim R)$
- $\sim P \vee Q$
- $Q \wedge \sim R$
- $\sim [P \vee \sim (Q \wedge R)]$

(13) Use DeMorgan's laws to rewrite each statement:

- It is false that she is crying and he is laughing.
- Neither is the cat strong, nor is the rabbit weak.
- He does not take a subway or he takes a bus.
- Either you have a ticket or you are going to buy one.

6. The conditional, valid arguments and Venn diagram

(1) For the conditional $P \to Q$,
- What other conditionals can be formed?
- Write down their truth tables;
- Which one of the conditionals is equivalent to $P \to Q$?

(2) Let $P =$ "I practise playing violin on Saturday," and $Q =$ "I play violin well." Write down in words:
- $P \to Q$:
- The converse of $P \to Q$:
- The inverse of $P \to Q$:
- The contrapositive of $P \to Q$:

(3) Let $P =$ "I exercise on Sunday," and $Q =$ "I am healthy." Write down in words:
- $P \to Q$:
- The converse of $P \to Q$:
- The inverse of $P \to Q$:
- The contrapositive of $P \to Q$:

(4) Present two examples of each of the following:

- If P, then Q $P \to Q$
- If P, Q $P \to Q$
- Q if P $P \to Q$
- P implies Q $P \to Q$
- Q is implied by P $P \to Q$
- Q whenever P $P \to Q$
- P only if Q $P \to Q$
- P is sufficient for Q $P \to Q$
- Q is necessary for P $P \to Q$

(5) Let P = "I study on Friday," and Q = "I will pass the exam." Write each of the following statements in symbolic form:
- If I study on Friday, then I will pass the exam.
- If I study on Friday, I will pass the exam.
- Passing the exam implies that I study on Friday.
- Not studying on Friday is implied by not passing the exam.
- I will pass the exam whenever I study on Friday.
- I do not study on Friday only if I will not pass the exam.
- Me studying on Friday is sufficient for me to pass the exam.
- Passing the exam is necessary for me to study on Friday.

(6) Biconditionals can be written in the form of "if and only if" or "necessary and sufficient." Give two examples of biconditionals and write them in both the forms.

(7) Use Venn diagrams to determine whether the arguments are valid:
-
  - No musician is lazy.
  - Some lazy individuals are rich.
  - Therefore, no musician is rich.
-
  - All students are industrious.
  - Some robots are industrious.
  - So, some robots are students.

- - All tables have four legs.
  - No person has four legs.
  - Therefore, no person is a table.

- - All dancers are strong.
  - Some singers are not dancers.
  - So, some singers are strong.

7. Definition of Probability and Sample Spaces

(1) Explain with an example the terms, *experiment*, *outcome*, *sample space*, *an event*, and *equally likely outcomes.*

(2) Suppose that you toss a die twice. What is the sample space of this experiment? What is the event that the sum of the numbers is at least 5? Are the outcomes equally likely?

(3) Explain with an example how you would calculate the probability of the occurrence of an event. What is the probability of the event in problem (2)?

(4) Present an example of an experiment and events of

- zero probability;

- perfect probability;

- probability $= \dfrac{3}{4}$

(5) Give an example of an experiment with an event with redundant outcomes. Calculate its probability.

(6) Give an example of an experiment with an event and its complement. Calculate their probabilities.

(7) This question is for group-work. Each group should have at least 4 students. You will perform six experiments. For each experiment, you will toss a die and record the outcome. The number of times you will toss the die are 1,5,10, 15, 15, and 25 respectively.

Toss	Outcome
1	

Experiment 1

Toss	Outcome
1	
2	
3	
4	
5	

Experiment 2

Toss	Outcome	Toss	Outcome
1		6	
2		7	
3		8	
4		9	
5		10	

Experiment 3

Toss	Outcome	Toss	Outcome	Toss	Outcome
1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10		15	

Experiment 4

Experiment 5

Toss	Outcome	Toss	Outcome	Toss	Outcome	Toss	Outcome
1		6		11		16	
2		7		12		17	
3		8		13		18	
4		9		14		19	
5		10		15		20	

Experiment 6

Toss	Outcome	Toss	Outcome	Toss	Outcome	Toss	Outcome	Toss	Outcome
1		6		11		16		21	
2		7		12		17		22	
3		8		13		18		23	
4		9		14		19		24	
5		10		15		20		25	

- Find the empirical probability of getting a 6 in each of the experiments.
- What is the theoretical probability of getting a 6 when you toss a die?

(8) This question is for group-work. Each group should have at least 4 students. You will perform six experiments. For each experiment, you will draw a card from a standard deck of cards, record the suit of the card, and replace the card. The number of times you will draw a card will be 1,5,10, 15, 15, and 25 respectively.

Draw	Outcome
1	

Experiment 1

Draw	Outcome
1	
2	
3	
4	
5	

Experiment 2

Draw	Outcome	Draw	Outcome
1		6	
2		7	
3		8	
4		9	
5		10	

Experiment 3

Draw	Outcome	Draw	Outcome	Draw	Outcome
1		6		11	
2		7		12	
3		8		13	
4		9		14	
5		10		15	

Experiment 4

Experiment 5

Draw	Outcome	Draw	Outcome	Draw	Outcome	Draw	Outcome
1		6		11		16	
2		7		12		17	
3		8		13		18	
4		9		14		19	
5		10		15		20	

Experiment 6

Draw	Outcome	Draw	Outcome	Draw	Outcome	Draw	Outcome	Draw	Outcome
1		6		11		16		21	
2		7		12		17		22	
3		8		13		18		23	
4		9		14		19		24	
5		10		15		20		25	

- Find the empirical probability of getting a diamond card in each of the experiments.
- What is the theoretical probability of getting a diamond card when you draw a card from a standard deck of cards?

(9) There are seven nickels, twelve dimes, four quarters, and eight dollar coins in a box. You pick a coin in random. What is the probability of getting

- a penny;
- a nickel;
- a dime;
- a quarter;
- a dollar;
- an amount greater than two cents?

(10) On a single toss of one die, find the probability of obtaining

- a ten;

- a factor of seven;
- a number divisible by three;
- a number divisible by 1.

(11) On drawing a single card from a standard deck of cards, find the probability of getting

- a spade;
- a spade or a Jack;
- a four of clubs;
- a four or a club;
- a picture card;
- a picture card of diamonds;
- a picture card or a diamond card.

(12) State the Fundamental Counting Principle. Explain using an example.

(13) Suppose that you toss a die and flip a coin. What is the sample space for this experiment? What is the probability of getting a Head or a six?

(14) Suppose that you toss two dice. What is the sample space for this experiment? What is the probability of getting

- at least one six;
- exactly one six;
- two sixes;
- a sum over seven;
- a sum under seven;
- a sum not over seven;
- a sum equal to seven?

(15) An experiment consists of flipping a coin and drawing a card from a standard deck of cards. What is the probability of getting

- a Head;
- a ten of diamonds;
- a Head and a ten of diamonds;
- a Head or a ten of diamonds?

(16) Forty-five people are surveyed regarding the sports they played. The responses were
- 22 played tennis;
- 25 swam;
- 23 played basket-ball;
- 12 both swam and played tennis;
- 13 both swam and played basket-ball;
- 11 played both tennis and basket-ball;
- 5 participated in all the three sports.

What is the probability that a randomly selected person
- does not participate in any of the sports;
- plays basket-ball but not the other two sports;
- swims or plays tennis?

(17) The table below shows the distribution of thirty-five students taking Mathematics or Biology, but not both.

	Mathematics	Biology
Male	13	7
Female	10	5

A student is chosen at random. What is the probability that the student is
- Female;
- Female and is taking Mathematics;
- Female, or is taking Mathematics;
- Male;
- Male and is taking Biology;
- Male, or is taking Biology?

8. Tree diagrams, Counting, Ordered Arrangements, and Permutations

(1) A family has three children. Use a tree diagram to list the sample space. What is the probability

that

- all the children are boys;
- there is exactly one girl;
- there is at least one girl?

(2) A box contains two one-dollar bills, two five-dollar bills, one ten-dollar bill, and a twenty-dollar bill. Two bills are chosen in succession without replacement. Use a tree diagram to list the sample space. What is the probability of getting

- exactly ten dollars;
- at least ten dollars;
- less than ten dollars;
- less than twenty dollars?

(3) How many five-lettered words can be formed from the letters A, B, C, D, E? (The words can be meaningless, for example, EBCDA).

(4) How many five-lettered words can be formed from the letters A, B, C, D, E if no letter is to be repeated?

(5) How many five-digit numbers can be formed from the digits $1, 2, 3, 4, 5$?

(6) How many five-digit numbers can be formed from the digits $1, 2, 3, 4, 5$ if no digit is to be repeated?

(7) $n! =$ ____.
$0! =$ ____.

(8) Calculate
$\bullet 5! =$ $\bullet 7! =$

(9) How many three-lettered words can be formed from the letters A, B, C, D, E if
- letters can be repeated?
- no letter can be repeated?

(10) How many three-digit numbers can be formed from the digits $1, 2, 3, 4, 5$ if
- digits can be repeated?
- no digit can be repeated?

(11) $nPr =$ ____.
$5P3 =$ ____.

(12) State the Fundamental Counting Principle. Explain using an example.

(13) How many social security numbers (SSNs) can be formed if
- the digits can be repeated in an SSN;
- the digits cannot be repeated in an SSN;
- the SSNs cannot start with a 0;

- the SSNs have to start with a 0?

(14) This year I plan to travel to Athens, Bombay, Chicago, and Dubai. I do not intend to visit any city more than once. How many possible itineraries are possible? How many itineraries are possible if I visit Dubai first?

(15) Given digits $0, 1, 2, 3, 4$, how many five-digit numbers can be formed if

- you are allowed to repeat the digits;
- you are not allowed to repeat the digits
- the numbers have to be even and the digits can be repeated;
- the numbers have to be odd and the digits cannot be repeated;

(16) Copy down the definition and the relevant formulae for permutations from pages 255 and 256 of your textbook.

(17) Calculate $\bullet 7P3$ $\bullet 8P8$ $\bullet 6P0$.

(18) How many distinct three-lettered words can be formed from the word EGG? Explain.

(19) How many distinct rearrangements can be formed from the letters of COOPER?

(20) How many distinct rearrangements can be formed from the letters of MISSISSIPPI?

9. Combinations and More Probability

(1) Write down the definition of a combination and the relevant formula from page 263 of your textbook.

(2) A certain committee has five members. In how many ways can

- a secretary, a chairperson, and an accountant be chosen, if no person can occupy two positions?
- a subcommittee of three members be chosen?

(3) From the set $\{A, B, C, D, E, F\}$, how many

- two-lettered words can be formed if no letter is to be repeated?
- two-element subsets can be formed?

(4) Write in your own words the difference between a permutation and a combination of r things chosen from n things.

(5) In how many ways can a hand of seven cards be chosen from a standard deck of cards?

(6) In how many ways can you choose seven cards one after another without replacement from a standard deck of cards?

(7) A hand of five cards is chosen from a standard deck of cards. What is the probability that

- all the cards are number cards;

- all the cards are picture cards;

- three cards are number cards, and two are picture cards;

- three cards are diamonds and two are spades?

(8) A committee consists of twelve men and fifteen women. A seven-person subcommittee is formed. What is the probability that

- all the seven are men;

- all the seven are women;

- three are men and four are women?

(9) A committee consists of twelve men and fifteen women. A chairperson, a deputy chairperson, a secretary, a deputy secretary, an accountant, a deputy accountant, and a stand-by member are to be chosen. What is the probability that

- all the seven are men;

- all the seven are women;

- both the secretaries are women, and the rest of the members are all men?

10. Measures of Central Tendency

(1) Write down the procedure of finding the mean for a given data from page 280 of your textbook.

(2) Make a list of five whole numbers and find its mean.

(3) Make a list of six whole numbers and find its mean.

(4) Explain using two different examples why the mean may not be a good measure of central tendency.

(5) Write down the procedure of finding the median for a given data from page 282 of your textbook.

(6) What is the median of the list from problem (2)?

(7) What is the median of the list from problem (3)?

(8) Explain using two different examples why the median may not be a good measure of central tendency.

(9) Write down the definition of mode, and a bimodal data from page 283 of your textbook.

(10) Do your examples in problems (2) and (3) have modes? If so, then what are they?

(11) Give a list of whole numbers with no mode.

(12) Give a list of whole numbers with a unique mode.

(13) Give a bimodal list of whole numbers.

(14) Give a list of whole numbers with more than two modes.

(15) Explain using examples why the mode may not be a good measure of central tendency.

(16) Write down the definition of the midrange from page 284 of your textbook.

(17) Find the midrange of your list from problem (2).

(18) Find the midrange of your list from problem (3).

(19) Using an example explain why the midrange may not be a good measure of central tendency.

(20) For the following lists find all the measures of central tendency:

- $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

- $300, 412, 520, 520, 670, 900, 1000, 20000$

(21) A student scores 70,82, 73, and 80 points in four in-class exams. What should be the student's final exam score, if his/her mean score is 81?

(22) The table below indicates the ages of a group of twenty people. Find all the measures of central tendency.

Age	Number of people
12	3
14	4
15	2
16	4
17	2
18	5

11. Measures of Dispersion and the Normal Curve

(1) Write down the description of the range from page 291 of your textbook.

(2) Make a list of ten whole numbers and finds its range.

(3) Explain using an example why the range may not be a good measure of dispersion

(4) Write down the procedure of calculating the standard deviation, σ, of a list of numbers.

(5) Calculate the standard deviation, σ, of your list from problem (2).

(6) Write down the characteristics of the standard deviation from page 295 of your textbook.

(7) For this problem, the words "small" and "large" are not well-defined. Use your intuition. Write down a list of five numbers with

- standard deviation = 0.

- small standard deviation.

- large standard deviation.

(8) Find the mean, median, mode, midrange, range, variance, and standard deviaiton for

- 4,5,5,8,9,10,10.

- 74,74,75,80,82,90.

(9) In an experiment, a die was tossed one hundred times. The outcomes are listed:

1	2	3	4	3	2	1	6	2	4
3	3	2	1	5	1	3	2	2	3
4	5	4	3	4	3	5	3	1	2
2	4	6	3	3	3	4	6	1	5
6	5	1	3	2	5	1	2	3	6

- Construct the frequency distribution.

- Construct its frequency polygon, and approximate the polygon by a smooth frequency distribution curve.

(10) Draw a frequency distribution curve such that its mode is less than its mean, with the median in between.

(11) Draw a frequency distribution curve such that its mode is greater than its mean, with the median in between.

(12) Draw a normal curve. State the main characteristics of the normal distribution from page 319 of your textbook.

(13) Read the summary on normal distributions from page 320 of your textbook. State some important points here.

(14) State the Empirical Rule (68-95-99.7 rule) from page 321 here.

(15) You will need Table 5.6 from page 322 for this problem. The heights of third graders at a certain elementary school are found to be approximately normally distributed with a mean (μ) of 45 inches and a standard deviation of 5 inches. What percentage of students are

- between 40 and 50 inches tall?

- at least 45 inches tall?

- at most 45 inches tall

- between 50 and 55 inches tall?

- less than 30 inches tall?

(16) Assume that the final scores of MTH 12 are normally distributed with a mean of 75 and a standard deviation of 10. What percentage of students score
- 93 points or higher?

- between 62 and 75 points?

- between 62 and 93 points?

12. Markups and Markdowns, and Simple Interest

(1) What percent of 32 is 16?

(2) What percent of 16 is 32?

(3) What percent of 81 is 900?

(4) What percent of 900 is 81?

(5) Find the dollar amount of markup, to the nearest cent:

Cost	Percentage markup on cost
\$ 50	4%
\$ 27	13%
\$ 125	$12\frac{1}{2}$%

(6) Find the markup and the percent markup on the cost and express each percent to the nearest hundredth.

Selling Price	Cost
\$ 50	\$ 30
\$ 100	\$ 80
\$ 48	\$ 15
\$ 60	\$ 45

(7) A watch retails for \$ 120.00. Find the cost of the watch if the markup is

- $33\frac{1}{3}$ % of the cost.

- 20 % of the retail price.

(8) A television cost \$ 80.00. What is its selling price if the markup is

- 40 % of the cost price?

- 35% of the selling price?

(9) State he Simple Interest formula from page 730 of your textbook. What is the formula for the final Amount?

(10) Find the simple interest and the final amount on a \$ 4000 loan at 9 % for

- 3 years

- 6 months

- 9 months

- 18 months

- 3 months

- 1 month

(11) Find the simple interest and the final amount on a \$ 4000 loan at $7\frac{1}{2}$ % for

- 3 years

- 6 months

- 9 months

- 18 months

- 3 months

- 1 month

(12) What is the simple interest and the final amount of a deposit of \$ 5000.00 at the end of two years if the rate of interest is 7%

- per annum;

- per month;

- per week;

(13) Find the missing quantities:

- P = $ 2000   r = 5% p.a.   t = 18 months   I =   A =

- P = $ 2000   r = 6% p.m.   t =   I =   A = $ 5000.00

- P =   r = 7% p.w.   t = 1 year   I =   A = $ 7000.00

13. Compound Interest and Life Insurance

(1) \$ 1000 is invested at 7%. Find

- the simple interest earned at the end of 3 years.

- the compound interest earned at the end of 3 years if the interest is compounded annually.

(2) Write down the formula for compound interest from page 737 of your textbook. State clearly what your variables stand for.

(3) Find the compound interest earned from \$ 1000 at the rate of interest of 6 % for 4 years if the interest is compounded

- yearly
- monthly
- weekly.

(4) Find the compound interest earned from \$ 1800 at the rate of interest of 6 % for 4 years if the interest is compounded

- yearly
- monthly
- weekly.

(5) Write a paragraph (at least 5 sentences) on Term Life Insurance.

(6) Write a paragraph (at least 5 sentences) on Endowment Policy.

(7) Write a paragraph (at least 5 sentences) on Permanent Life Insurance.

(8) Write a paragraph (at least 5 sentences) on Whole Life Insurance. What are the straight life and limited payment life insurance policies?

(9) Write a paragraph (at least 5 sentences) on Universal Life Insurance.

(10) Write a paragraph (at least 5 sentences) on Variable Life Insurance.

(11) Write a paragraph (at least 5 sentences) on Variable-Universal Life Insurance.

(12) Write a paragraph (at least 5 sentences) on how Insurance premiums are determined.

(13) You will need Table 12.2 of your textbook for this problem. Find the annual premium for each of the given insurance policy.

Face value of policy	Age at issue	Type of policy
$ 40,000	35	5-year term
$ 35,000	40	Straight life
$ 28,000	45	Limited payment (20-year)
$ 12,000	50	Endowment (20-year)

(14) You wil need Tables 12.2 and 12.3 for this problem. For the same table as in problem (13), determine the annual cost if the payment is made

- semiannually
- quarterly
- monthly.

(15) Rob is 35 years old and purchases a 5-year term life insurance policy with face value of $ 12,000. At the end of the term, he renews the policy. What is the difference in total cost between his first and second policies?

14. Installment Buying and Mortgages

(1) Suppose that I borrow \$ 6000 from a credit card company at 15% interest for six months. Then

- Calculate the interest accrued at the end of 6 months.
- So, the amount to be repayed by the end of 6 months is
- Suppose the credit card company requires me to repay in installments every month, then how much do I pay every month?
- Of this, ____________ is paid towards the principal, and ____________ is paid towards the interest.
- Is this truly fair? For instance,

The amount owed for the first month	=
The amount owed for the second month	=
The amount owed for the third month	=
The amount owed for the fourth month	=
The amount owed for the fifth month	=
The amount owed for the sixth month	=
New total amount owed for one month	=

- Since the total interest paid = ____________, for one month, the True Annual Rate of interest (also known as the Annual Percentage Rate, or the APR) is calculated as follows:

(2) Write down the formula for the APR from page 753 of your textbook.

(3) Use the APR formula to find the APR in the above situation.

(4) Mary borrowed \$ 8000 on the installment plan. She made a down payment of \$ 800, and agreed to pay the balance in 36 equal monthly payments at 15% simple interest.

- What was the amount financed?
- What is the total interest to be paid?
- What is each payment equal to?
- What is the total amount paid?
- What is the APR?

(5) I bought a used car for \$ 7000 which was financed at \$ 400 per month for two years. There was no down payment required. Find,

- the total price;
- the finance charge;
- the simple interest rate;
- the Annual Percentage rate.

(6) Write a paragraph (at least 5 sentences) on mortgage.

(7) Write a paragraph (at least 5 sentences) on Amortization.

(8) You will need Table 12.5 for this problem. Suppose I obtain a \$ 250,000 mortgage for 20 years at 6%.

- What is my monthly payment?
- How much total mortgage interest will I pay?

(9) You will need Table 12.5 for this problem. Suppose I obtain a \$ 250,000 mortgage for 10 years at 7%.

- What is my monthly payment?

- How much total mortgage interest will I pay?

(10) Suppose I purchase a \$ 120,000 apartment and pay a down payment of 15%. Bank A offers me a 30-year mortgage at 8%. Bank B offers me a 20-year mortgage at 9%. Which one is better if I want to minimize the amount of interest that I pay?

Bank A:

Monthly payment :

Number of payments :

Total payment :

Total interest :

Bank B:

Monthly payment :

Number of payments :

Total payment :

Total interest :

15. Practice problems

(1) A snowblower costs a retailer \$310. The markup is 40% of the cost. Find the selling price, to the nearest cent.

(2) An airconditioner costs a retailer \$310. The markup is 40% of the selling price. Find the selling price, to the nearest cent.

(3) Find the following:
(a) Simple interest earned on a \$2,500 deposit for 3 years at 11%.
(b) Compound interest earned on a \$2,500 deposit for 3 years at 11% compounded monthly.

(4) A videocassette recorder is advertised for \$600. It may be purchased on the installment plan by paying \$50 and agreeing to pay the balance plus 18% simple interest on the balance in 24 monthly payments.
(a) What is the finance charge?
(b) What is the amount of each payment?
(c) What os the total cost of the videocassette recorder?
(d) What is the true annual interest rate or APR?

(5) The Smiths assumed a \$40,000 mortgage for 20 years at 10.5%. What is their monthly payment? How much total interest will the Smiths pay on their mortgage?

(6) The set of data $\{2,7,4,8,10,7,6,12\}$ is given. Find
(a) mean (b) mode (c) median (d) midrange (e) range
(f) standard deviation.

(7) The mean of a set of 5 scores is 86, what is the sum of 5 scores?

(8) If the variance for a set of data is 49, what is the standard deviation?

(9) Use the provided Table for the standard Normal Distribution curve to answer the following: (a) $\Pr(Z \geq 1.1)$ (b) $\Pr(-2.11 \leq Z \leq 0.05)$ (c) $\Pr(0.12 \leq Z \leq 1.1)$.

(10) Fill in the blank.
Z-score for data X is given by $Z = \dfrac{X-\mu}{\sigma}$ where $\mu =$ mean and $\sigma =$

(11) The height of students at a local high school are approximately normally distributed with a mean height 68 inches and a standard deviation of 3 inches. Use Table E to find
(a) the percentage of students between 66 inches and 72 inches tall
(b) the percentage of students between 70 inches and 72 inches tall
(c) the percentage of students over 6 feet tall (1 foot = 12 inches)
(d) the percentage of students shorter than 5 feet 6 inches tall
(e) the percentage of students between 5 feet 6 inches and 6 feet tall

(12) On a single toss of one die, find the probability of obtaining
a) an odd and even number
b) a number less than and equal 3
c) an odd or an even number
d) a number divisible by 4
e) a number is a multiple of 3.

(13) A box contains a five-dollar bill, a ten-dollar bill, a twenty-dollar bill, a fifty-dollar bill and a hundred-dollar bill. Two bills are chosen at random in succession without replacing the first bill

before the second is drawn.

a) How many outcomes are possible?

b) What is the probability that the value of the bill is even?

c) What is the probability that the value of the second bill is even?

d)What is the probability that the values of the both bills are even?

e) what is the probability the sum of the value of two bill is \$ 35?

(14) In a survey of 45 contestants at racquetball tournament, the following information is obtained: 14 contestants entered both the singles and doubles tournaments, 27 contestants were entered in the singles tournament, 20 entered the double tournament.

a) Display the results of the survey in a Venn Diagram.

b) Find the probability that a contestant is entered in the single tournament only.

c) Find the probability that a contestant is entered in neither single nor the double tournament.

(15) A New York State car license plate consists of a letter, 3 digits followed by 2 letters. For example, T123KI.

a) How many of these car license plates can be formed if no digit and letter can be repeated?

b) How many of these car license plates can be formed if only repetition of letters and no repetition of digit are allowed?

(16) A four-digit numbers is formed from the set of digits {2,5,3,7,9,6} without repetition of digit

a) How many four-digit numbers can be formed?

b) How many of these will be odd?

c) How many will be divisible by 5?

(17) Evaluate the following:

a) $\dfrac{7!}{4!3!}$ b) ${}_7P_3$ c) ${}_8C_6$

(18) In how many ways can a newspaper boy deliver 5 newspapers to 6 houses?

(19) At registration, a student needs three more courses to complete her schedule. If there are five possible courses left to pick from, in how many ways can she choose the two courses?

(20) Don has to take an English exam that consists of 10 multiple-choice questions and 5 essay questions. If Don has to answer 7 multiple questions and 3 essay questions, in how many ways can he choose them?

(21) Fill in the blank.

Type of Statement	Connective	Symbol
Negation	not	$\sim$
		$\vee$
	and	
Conditional		
		$\longleftrightarrow$

(22) Let $P =$ *Algebra is difficult*, $Q =$ *Logic is easy, and* $R =$ *Latin is interesting.* Use the appropriate connectives and parentheses to symbolize each statement.

a) If logic is easy and algebra is not difficult, then Latin is interesting or algebra is not difficult.

b) Algebra is not difficult iff Latin is not interesting and logic is easy.

c) It is false that Logic is not easy and algebra is difficult.

d) Logic is easy only if algebra is not difficult.

e) If Latin is interesting or algebra is difficult, then it is false that logic is easy or Algebra is difficult.

(23) Let $P =$ *Algebra is difficult,* $Q =$ *Logic is easy, and* $R =$ *Latin is interesting.* Write each symbolic statement in words.

a) $P \vee (\sim Q \vee R)$

b) $\sim (\sim P \wedge \sim Q)$

(24) Construct a truth table for each symbolic statement.

a) $\sim P \rightarrow \sim Q$ b) $(P \wedge Q) \rightarrow Q \vee \sim P$

(25) Use De Morgan's Law to rewrite the sentence : It is false that Pam did not stay late or Angie left early.

(26) Symbolize each of the following arguments (using the suggested notation) and, by means of a truth table, determine whether the argument is valid or invalid. State your answer.

a) It will be neither sunny nor cloudy today (S, C)
It isn't sunny.
Therefore, it will be cloudy

b) Addie and Bill will not be at the party. (A, B)
Bill was at the party.
Therefore, Addie was at the party.

(27) Use De Morgan's Law to rewrite the sentence : It is false that Pam stayed late or Angie left early.

(28) Let $P =$ *Algebra is difficult,* $Q =$ *Logic is easy, and* $R =$ *Latin is interesting.* Use the appropriate connectives and parentheses to symbolize each statement.

a) If logic is easy and algebra is difficult, then Latin is interesting.

b) Algebra is difficult iff Latin is not interesting and logic is easy.

c) It is false that Logic is easy and algebra is not difficult.

d) Logic is easy only if algebra is difficult.

e) If Latin is not interesting or algebra is difficult, then logic is easy

(29) Let $P =$ *Algebra is difficult,* $Q =$ *Logic is easy, and* $R =$ *Latin is interesting.* Write each symbolic statement in words.

a) $P \wedge (Q \vee R)$

b) $\sim P \wedge \sim Q$

(30) Construct a truth table for each symbolic statement.

a) $P \rightarrow \sim Q$ b) $P \rightarrow Q \vee \sim P$

(31) Fill in the blank.

Type of Statement	Connective	Symbol
Negation	not	$\sim$
Disjunction		
Conjunction		
Conditional		
Biconditional		

(32) Symbolize each of the following arguments (using the suggested notation) and, by means of a truth table, determine whether the argument is valid or invalid. State your answer.

a) It will be raining or cloudy today (R, C)
It isn't cloudy.
Therefore, it will be raining

b) Sue and Bill will be at the party. (A, B)
Bill was at the party.
Therefore, Sue was at the party.

(33) Let $P = $ *Bill is golfing*, $Q = $ *Addie is sailing*, $R = $ *Flo is jogging*. be statements. Write each statement in words.

(a) $P \wedge (Q \vee R)$
(b) $(P \wedge Q) \vee R$
(c) $P \vee (Q \longrightarrow R)$

(34) By means of the appropriate connectives and parentheses, symbolize each statement, using the given symbols for the simple statements: $P = $ *Bill is golfing*, $Q = $ *Addie is sailing*, $R = $ *Flo is jogging*.

(a) If Bill is not golfing, then Addie is not sailing or Flo is jogging.
(b) Either it is false that both Bill is golfing and Flo is not jogging, or Addie is sailing.
(c) Addie is not sailing if and only if Bill is golfing or Flo is not jogging

(35) True or False.

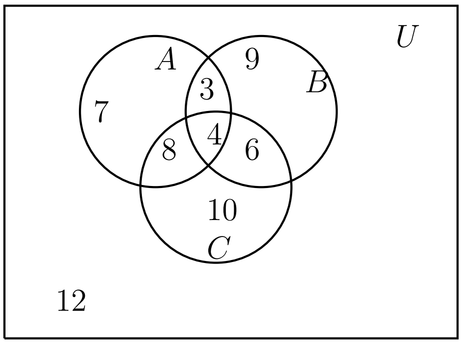
a. $99 \in \{3, 6, 9, 12, 15, ...\}$
b. $\emptyset \in \{\}$
c. $\{l, o, v, e\}$ is equivalent to $\{h, a, t, e\}$
d. $\{x | 3x + 3 = 9\} = \{2\}$
e. $\{1, 8\} \subseteq \{1, 2, 3, ...\}$
f. The set of well-behaved children is a well-defined set.
g. $\{a, b, c, d\} \cap \{b, d, f, h, j\} = \{a, b, f, h, j\}$
h. $\{a, b, c, d, d, d, e, f\}$ is equivalent to $\{a, b, c, d, e, f\}$.
i. $\emptyset$ is a finite set.
j. The set of natural numbers is a finite set.

(36) List any three subsets of $\{t, h, r, e\}$

(37) Let $U = \{0, 1, 2, 3, 4, 5, 6, 7\}, A = \{0, 1, 3, 5, 7\}, B = \{0, 2, 4, 6\}$, and $C = \{1, 3, 5, 7\}$. Find the each of the following.

a. $A \cap B =$
b. $B \cup C=$
c. $A' \cap B'=$
d. $(A \cup B)' =$
e. $(A \cup C')' \cap \emptyset =$

(38) Use the figure below to find each cardinality (= the total number of elements).



a. $n(A) =$
b. $n(A \cap B) =$
c. $n(A' \cap C)=$
d. $n(A \cup B)' =$
e. $n(U)=$

(39) In certain Las Vegas casino, a survey of 125 gamblers was taken and the following data were collected: 71 played roulette, 72 played poker, and 80 played blackjack, while 33 played roulette and poker, 42 played roulette and blackjack, 47 played poker and blackjack. Eleven played all three games.

a. Complete a Venn diagram with correct number in each region.
b. How many of these gamblers played only blackjack?
c. How many of these gamblers played poker and roulette, but not blackjack?
d. How many of these gamblers did not play poker?

(40) Use the figure below to find each cardinality (= the total number of elements).

a. $n(A) =$
b. $n(A \cap B) =$
c. $n(A' \cap C)=$
d. $n(A \cup B)' =$
e. $n(U)=$

(41) List any five subsets of $\{A, B, F, K, V\}$

(42) Let $U = \{1, 2, 3, 4, 5, 6, 7\}, A = \{1, 7\}, B = \{2, 4, 6\}$, and $C = \{3, 5, 7\}$. Find the each of the following.

a. $A \cap B =$
b. $B \cup C=$
c. $A' \cap B'=$
d. $(A \cup B)' =$
e. $(A \cup C')' \cap B =$

(43) True or False.

a. $-18 \in \{3, 6, 9, 12, 15, ...\}$
b. $\emptyset \in \{\emptyset\}$
c. $\{l, o, v, e\} = \{h, a, t, e\}$

d. $\{x|3x+3=9\}=\{3\}$

e. $\{1,8\} \subseteq \{1,8\}$

f. The set of married couples is a well-defined set.

g. $\{a,b,c,d\} \cap \{b,d,f,h,j\} = \{b,d\}$

h. $\{a,a,b,c,d,d,d,e,f\}$ is equivalent to $\{a,b,c,d,e,f\}$.

i. $\emptyset$ is a finite set.

j. The set of integers is an infinite set.

(44) In certain Las Vegas casino, a survey of 155 gamblers was taken and the following data were collected: 76 played roulette, 72 played poker, and 85 played blackjack, while 37 played roulette and poker, 42 played roulette and blackjack, 47 played poker and blackjack. Ten played all three games.

a. Complete a Venn diagram with correct number in each region.

b. How many of these gamblers played only blackjack?

c. How many of these gamblers played poker and roulette, but not blackjack?

d. How many of these gamblers did not play poker?

e. How many of these gamblers played poker or roulette?

(45) 1. True or False

a. $14 \in \{2,4,6,8,...\}$

b. $0 \in \{\}$

c. $\{\} \subseteq \{0\}$

d. $\{x \mid x+3=9\} = \{3,9\}$

e. $\{A,B,C\}$ is equivalent to $\{a,b,c\}$

f. The set of 5 elements has the total number of 32 subsets

2. If $U=\{m,e,t,r,i,c\}$, find

a. $\{m,e,t\}'$

b. $\{r,e,c,t,i\}'$