# BRONX COMMUNITY COLLEGE of the City University of New York

## DEPARTMENT OF MATHEMATICS & COMPUTER SCIENCE

### MTH06 Review Sheet

**1.** Perform the indicated operations and simplify:

(a) 
$$\frac{n^2 - 3n - 10}{n^2 + n - 2}$$
 (b)  $\frac{1 + \frac{5}{3y}}{\frac{6}{2x} + \frac{5}{y}}$  (c)  $\frac{2x^2 - x}{2x^2 + x - 1}$   
(d)  $\left(\frac{x^2 - 2xy - 3y^2}{x^2 - 9y^2}\right) \left(\frac{2x^2 - xy}{2x^2 + xy - y^2}\right)$  (e)  $\frac{9ab}{3a + b} \div \frac{a^2 - 6a}{a^2 - 4a - 12}$  (f)  $\frac{10}{x^2 - 5x} + \frac{2}{x}$   
(g)  $\frac{3}{n^2 - 5n - 36} - \frac{2}{n^2 + 3n - 4}$  (h)  $\frac{1}{x - 3} - \frac{3}{3 - x}$  (i)  $\frac{1}{6x(x - 2)^2} - \frac{1}{4x^2(x - 2)}$ 

**2.** Solve:

(a) 
$$\frac{1}{2x-7} + \frac{x-5}{4x^2-49} = \frac{4}{6x-21}$$
 (b)  $\frac{4}{5y-3} = \frac{2}{3y+7}$  (c)  $n + \frac{1}{n} = \frac{53}{14}$ 

(d) 
$$2 + \frac{4}{x-2} = \frac{8}{x^2 - 2x}$$

**3.** Simplify (express results with positive exponents only and rationalized denominators):

(a) 
$$16^{-3/4}$$
 (b)  $(4x^5y^{-2})^{-2}$  (c)  $\frac{4\sqrt{12}}{\sqrt{5}}$  (d)  $\sqrt{\frac{3}{8}}$ 

(e) 
$$\sqrt[3]{\frac{27}{64}}$$
 (f)  $\sqrt[3]{81x^3y^4z^7}$  (g)  $\left(-8x^{-6}y^{12}\right)^{1/3}$ 

4. Perform the indicated operations and simplify:

(a) 
$$3\sqrt{20} - \sqrt{5} - 3\sqrt{45}$$
 (b)  $\sqrt[3]{3} + 2\left(\sqrt[3]{24}\right) - 6\left(\sqrt[3]{81}\right)$  (c)  $-4\sqrt{2}\left(2\sqrt{5} - 3\sqrt{6}\right)$ 

(d) 
$$\left(2\sqrt{6} + 3\sqrt{5}\right)\left(\sqrt{8} - 3\sqrt{12}\right)$$
 (e)  $\frac{\sqrt{7}}{3\sqrt{2} - 5}$  (f)  $\frac{\sqrt{12}}{4} + \frac{\sqrt{27}}{6}$ 

**5.** Solve for x and check your solutions:

(a) 
$$\sqrt{2x-5} = 5$$
 (b)  $\sqrt{4x+2} = \sqrt{3x+4}$  (c)  $4\sqrt{x}+5 = x$  (d)  $\sqrt{x+1} - \sqrt{2x} = 1$ 

(a) Solve by factoring:  $x^2 - 7x + 12 = 0$ 

(b) Solve with the quadratic formula:  $4x^2 + 2x + 1 = 0$ 

- (c) Solve by completing the square:  $2x^2 8x 3 = 0$
- (d) Solve by any method:  $x^2 + 10x + 26 = 0$

7. Simplify (express your results in the form a + bi for a, b real):

(a) (4-8i) - (8-3i) (b)  $(\sqrt{-4}) (\sqrt{-16})$  (c) 7i (-9+3i)(d) (10+2i)(-2-i) (e)  $\frac{-1-3i}{4-5i}$  (f)  $2(\cos 120^\circ + i \sin 120^\circ)$ 

8. Given  $f(x) = 3x^2 - 2$ , determine:

(a) 
$$f(2)$$
 (b)  $f(-3)$  (c)  $f(1/2)$  (d)  $f(a)$ 

**9.** Sketch the graphs of each of the given functions, indicating the x and y intercepts, the vertex, the axis of symmetry and stating the maximum or minimum value of the function:

(a) 
$$f(x) = x^2 - 3$$
  
(b)  $g(x) = -2x^2 + 2$   
(c)  $h(x) = -(x+2)^2$   
(d)  $k(x) = -(x+2)^2 + 2$   
(e)  $w(x) = x^2 + x + 1$ 

**10.** Given  $f(x) = \frac{x^2 - 9}{x^2 + 2x - 15}$ 

(a) Determine the values of x for which the function is defined

- (b) Evaluate: f(0)
- (c) Evaluate: f(-3)

**11.** Graph:

(a) 
$$2x + 3y \le 6$$
 (b)  $x - 2y < 4$ 

12. Given the functions f(x) = 2x + 1 and  $g(x) = \frac{1}{2}x - \frac{1}{2}$  sketch both graphs on the same set of axes.

13. Sketch each pair of functions on the same set of axes:

(a) 
$$f(x) = 3^x$$
 and  $g(x) = 3^{-x}$  (b)  $f(x) = 3^x$  and  $g(x) = \log_3 x$ 

14. Solve for x (use the definitions and properties of exponents and logarithms):

(a)	$3^{x-1} = 81$	(b)	$7^{-x} = 49$	(c)	$25^x = 125$
(d)	$\log_4 1 = x$	(e)	$\log_6(6^{-8}) = x$	(f)	$\log_2 x = -4$

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15. Solve and write the answer in set notation:

(a) 
$$|2x-1| = 3$$
 (b)  $|x|-1 = 4$  (c)  $|2x-1| \le 3$  (d)  $|x| > 5$  (e)  $|2-3x| \ge 1$ 

16. Find the center and radius for each circle:

(

a) 
$$x^2 + y^2 - 4y = 12$$
 (b)  $x^2 + y^2 = 9$  (c)  $x^2 - 6x + y^2 - 4y = 8$ 

17.

- (a) Find  $\theta$ , to the nearest whole degree, if  $\sin \theta = 0.1234$  and  $\cos \theta < 0$
- (b) Find  $\sin \theta$  if  $\cos \theta = -3/5$  and  $\theta$  is in Quadrant III.

**18.** Without using a calculator, determine the exact value of:

(a) 
$$\sin^2(30^\circ) + \cos^2(30^\circ)$$
 (b)  $\tan(60^\circ)$  (c)  $\sec^2(45^\circ) - \tan^2(45^\circ)$  (d)  $\sin(\pi/6) - \cos(\pi/4)$ 

**19.** If each of the following points P are on the terminal side of angle  $\theta$  in standard position with  $0 \le \theta < 360^{\circ}$ , draw  $\theta$  and determine the value of the six trigonometric functions of  $\theta$ :

(a) 
$$P = (3, -2)$$
 (b)  $P = (-3, 4)$  (c)  $P = (2, 4)$ 

**20.** For each of the following angles  $\theta$ , draw them in standard position, choose a specific point on the terminal side of  $\theta$  and determine the exact values of  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  without using a calculator:

(a) 
$$\theta = \frac{5}{6}\pi$$
 (b)  $\theta = 315^{\circ}$  (c)  $\theta = 270^{\circ}$ 

**21.** Evaluate  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  exactly for each of the following angles:

(a) 
$$\theta = 210^{\circ}$$
 (b)  $\theta = -240^{\circ}$  (c)  $\theta = 675^{\circ}$ 

**22.** Solve the following (clearly specify the unknown, draw a labeled diagram if appropriate and state the solution in words):

- (a) The time a person takes to paddle a kayak 2 miles downstream is the same as the time to paddle half a mile upstream. If the rate of the current is 3 mph, what is the person's paddling rate in still water?
- (b) Bill is standing on top of a 175 foot cliff overlooking a lake. The measure of the angle of depression to a boat is 29°. How far is the boat from the bottom of the cliff (rounded to one decimal place)?
- (c) Suppose that the height in meters of a golf ball, hit from a tee, is approximated by  $y = -5t^2 + 10t + 15$  where t is the time in seconds. Find the maximum height of the ball and the time it reaches this maximum height.
- **23.** Graph each equation for  $-2\pi \le x \le 2\pi$ :

(a)  $y = 2\sin x$  (b)  $y = 3\cos x$  (c)  $y = -\cos x$ 

24. A central angle of  $\theta = 30^{\circ}$  is contained in a circle with radius r = 30 inches. Find (leaving all results in terms of  $\pi$ ):

- (a) the length of the arc subtended by  $\theta$
- (b) the area of the sector determined by  $\theta$



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25. Find the exact values of the area and perimeter of this right triangle:



- 26. Verify the following trigonometric identities:
- (a)  $\csc \theta \tan \theta \cos \theta = 1$
- (b)  $\csc \theta \sin \theta = \cot \theta \cos \theta$
- (c)  $\tan^2 \theta + 1 = \sec^2 \theta$

(d) 
$$\cos^2 \theta - \sin^2 \theta = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

#### 27.

1.

- (a) A hot-air balloon rises vertically. An observer stands on level ground at a distance of 125 feet from a point on the ground directly below the passenger's compartment. How far, to the nearest foot, does the balloon rise if the angle of elevation changes from 20° to 30°?
- (b) A state trooper is hiding 30 feet from a straight highway with a speed limit of 65 mph. One second after a truck passes, the angle  $\theta$  between the highway and the line of observation from the patrol car to the truck is measured.
  - (i) If  $\theta = 15^{\circ}$  does the truck driver get a speeding ticket (1 mile = 5,280 ft)?
  - (ii) If  $\theta = 30^{\circ}$  does the truck driver get a speeding ticket?

#### Answers

(a) 
$$\frac{n-5}{n-1}$$
 (b)  $\frac{3xy+5x}{3(3y+5x)}$  (c)  $\frac{x}{x+1}$  (d)  $\frac{x}{x+3y}$  (e)  $\frac{9ab+18b}{3a+b}$   
(f)  $\frac{2}{x-5}$  (g)  $\frac{n+15}{(n-9)(n+4)(n-1)}$  (h)  $\frac{4}{x-3}$  (i)  $\frac{-x+6}{12x^2(x-2)^2}$   
2.  
(a)  $x = 22$  (b)  $y = -17$  (c)  $n = 2/7, 7/2$  (d)  $x = -2$   
3.  
(a)  $\frac{1}{8}$  (b)  $\frac{y^4}{16x^{10}}$  (c)  $\frac{8\sqrt{15}}{5}$  (d)  $\frac{\sqrt{6}}{4}$  (e)  $\frac{3}{4}$  (f)  $3xyz^2\sqrt[3]{3yz}$  (g)  $\frac{-2y^4}{x^2}$   
4.  
(a)  $-4\sqrt{5}$  (b)  $-13\sqrt[3]{3}$  (c)  $-8\sqrt{10}+24\sqrt{3}$   
(d)  $8\sqrt{3}-36\sqrt{2}+6\sqrt{10}-18\sqrt{15}$  (e)  $-\frac{3\sqrt{14}+5\sqrt{7}}{7}$  (f)  $\sqrt{3}$ 



(a)  $(-\infty, -5) \cup (-5, 3) \cup (3, \infty)$  or all real numbers except -5 and 3 (b) f(0) = 3/5 (c) f(-3) = 0









(a) x = 5 (b) x = -2 (c) x = 3/2 (d) x = 0 (e) x = -8 (f) x = 1/16

(a)  $\{-1,2\}$  (b)  $\{-5,5\}$  (c)  $\{x \mid -1 \le x \le 2\}$  (d)  $\{x \mid x < -5 \text{ or } x > 5\}$  (e)  $\{x \mid x \le 1/3 \text{ or } x \ge 1\}$ 16.

- (a) center (0,2) radius 4 (b) center (0,0) radius 3 (c) center (3,2) radius  $\sqrt{21}$
- 17.

(a)  $\theta = 173^{\circ}$  (b)  $\sin \theta = -4/5$ 

18.

(a) 1 (b)  $\sqrt{3}$  (c) 1 (d)  $1/2 - \sqrt{2}/2 = (1 - \sqrt{2})/2$ 

19.





(a) 
$$\sin 210^\circ = -\sin 30^\circ = -1/2$$
,  $\cos 210^\circ = -\cos 30^\circ = -\sqrt{3}/2$ ,  $\tan 210^\circ = \tan 30^\circ = \sqrt{3}/3$ 

(b) 
$$\sin(-240^\circ) = \sin 60^\circ = \sqrt{3}/2$$
,  $\cos(-240^\circ) = -\cos 60^\circ = -1/2$ ,  $\tan(-240^\circ) = -\tan 60^\circ = -\sqrt{3}$ 

(c) 
$$\sin 675^{\circ} = -\sin 45^{\circ} = -\sqrt{2}/2$$
,  $\cos 675^{\circ} = \cos 45^{\circ} = \sqrt{2}/2$ ,  $\tan 675^{\circ} = -\tan 45^{\circ} = -1$ 

22.

(a) The person's paddling rate in still water is 5 mph.

(b) The boat is 315.7 feet from the bottom of the cliff.

(c) The maximum height of the ball is 20 meters which it reaches in 1 second.

23.





(a) arc length =  $5\pi$  inches (b) area =  $75\pi$  square inches

#### 25.

Area =  $2 \tan 30^{\circ}$  in<sup>2</sup> =  $\frac{2\sqrt{3}}{3}$  in<sup>2</sup>; perimeter =  $2\sqrt{3} + 2$  in

#### 26.

To prove these identities, use algebra and the basic identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$
$$\cos^2 \theta + \sin^2 \theta = 1$$
$$\csc \theta = \frac{1}{\sin \theta}$$
$$\sec \theta = \frac{1}{\cos \theta}$$
$$\cot \theta = \frac{1}{\tan \theta}$$

27.

- (a) The balloon rises 27 feet.
- (b) (i) The truck is traveling at 111.96 ft/sec which is 76.34 mph and the driver gets a ticket. (ii) The truck is traveling at 51.96 ft/sec which is 35.43 mph and the driver does not get a speeding ticket.

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