Ratios and Proportions

At a certain college, there are 500 students who are under 25 years of age and 1500 students who are 25 years old or older. The ratio of students under 25 to students over 25 is 500 to 1500, which is also written 500:1500 or as a fraction $\frac{500}{1500}$. We know that as fractions $\frac{500}{1500} = \frac{1 \cdot 500}{3 \cdot 500} = \frac{1}{3}$. A ratio is a division relationship between two numbers.

The ratio of a to b, written as a:b, is another meaning for the division or fraction $\frac{a}{b}$.

Equivalent ratios: We have already noticed that the ratios $\frac{500}{1500} = \frac{1}{3}$ are equal as fractions. Notice too that $500 \cdot 3 = 1 \cdot 1500$. Two ratios are equal if and only if their cross products are equal. $\frac{a}{b} = \frac{c}{d}$ if and only if $a \cdot d = c \cdot b$. Observe how the cross products are constructed. The numerator on the left side times the denominator on the right is equal to the numerator on the right times the denominator on the left.

Proportions: An equation of two ratios is called a proportion. $\frac{500}{1500} = \frac{1}{3}$ is a proportion, and $\frac{64}{360} = \frac{16}{90}$ is a proportion.

A proportion involves 4 numbers. We can use cross-multiplication and some algebra to solve a proportion equation for an unknown.

Example 1: Find t so that this proportion is true.

$$\frac{140}{210} = \frac{8}{t}$$
Set the cross products equal and solve the equation.
$$140t = 8 \cdot 210$$
$$140t = 1680$$
$$t = \frac{1680}{140}$$
$$t = 12$$

Proportions can be used to solve many word problems.

Example 2: How far does a train going 68 miles per hour travel in 3 and a half hours?
$$\frac{68 \text{ miles}}{1 \text{ hour}} = \frac{x \text{ miles}}{3.5 \text{ hours}}$$
$$68 \cdot 3.5 = x \cdot 1$$
$$x = 238 \text{ miles}$$

Answers to odd exercises on reverse: 1. x = 25; 3. w = 60; 5. x = $\frac{200}{3}$; 7. P = 140; 9. original price = $40, sale price = $30; 11. 3.2 inches

Percents and Proportions: Percent means parts out of 100 parts, so 25% is equal to $\frac{25}{100}$.

A percent relationship, such as 30 is 25% of 120, can be written as a proportion:

$\frac{30}{120} = \frac{25}{100}$. Here 30 is called the amount, 120 is the base, and 25 is the percent. The

basic percent relationship is $\frac{\text{amount}}{\text{base}} = \frac{\text{percent}}{100}$. This percent proportion can be

used to find the third number in a percent relationship when the other two numbers are known.

Example 3: 200 is 80% of what number?

$\frac{200}{b} = \frac{80}{100}$ Let b represent the unknown number which is the base in the percent relationship and set up the proportion.

$\frac{200}{b} = \frac{4}{5}$ Cross-multiply and solve. It helps to reduce the ratio

$4b = 200 \cdot 5$ first.

$b = \frac{1000}{4} = 250$

Exercises:

In exercises 1-8, find the value of the variable that makes the proportion true.

1. $\frac{10}{18} = \frac{x}{45}$

2. $\frac{5}{8} = \frac{25}{y}$

3. $\frac{120}{200} = \frac{36}{w}$

4. $\frac{1}{8} = \frac{t}{6}$

5. $\frac{x}{10} = \frac{200}{30}$

6. $\frac{500}{25} = \frac{100}{y}$

7. $\frac{7}{5} = \frac{P}{100}$

8. $\frac{A}{75} = \frac{15}{100}$

9. Elisa bought a blouse on sale. She saved 25% of the original price, or $10. What was the original price of the blouse? How much did she pay for the blouse?

10. If a car gets 30 miles per gallon of gas, how many gallons of gas are needed to travel 345 miles?

11. On a map, one inch represents 50 kilometers. How may inches represent 160 kilometers?