Glossary

Absolute value The *absolute value* of a number is its magnitude. It is represented symbolically by two bars around a number $|\ \cdot\ |$.

Addition principle The *addition principle* expresses the fact that when the same quantity is added to (or subtracted from) each side of an algebraic statement, the resulting statement is equivalent to (has the same solutions as) the original one.

Algebraic expression An *algebraic expression* is an expression formed by combining numbers and variables using the operations of addition, subtraction, multiplication, division, (numerical) exponents, and roots.

Algebraic statement An *algebraic statement* is a comparison of two algebraic expressions using the relations of equality, greater than, or less than. An algebraic statement may be true or false.

Associative law The *associative law* (of addition or of multiplication) expresses the fact that when performing the same operation several times, the way in which quantities are grouped, and hence the order that the operations are performed, does not affect the outcome of performing the operations. It is usually expressed symbolically as $(a+b)+c = a+(b+c)$ (for addition) or $(a \cdot b) \cdot c = a \cdot (b \cdot c)$ (for multiplication).

Base (of an exponential expression) In an exponential expression, the *base* is the quantity which is written immediately to the left of the exponent (which is written as a superscript). For natural number exponents, the base is the quantity which is multiplied a repeated number of times.

Coefficient The *coefficient* of a term is the "number part" of the term, or the constant by which the variable part is multiplied.

Commutative law The *commutative law* (of addition or of multiplication) expresses the fact that the order in which the quantities are expressed does not affect the outcome of performing the operation. It is usually expressed symbolically as $a + b = b + a$ (for addition) or $a \cdot b = b \cdot a$ (for multiplication).

Completing the square *Completing the square* is a process by which an appropriate quantity is added to an algebraic expression in such a way that the result is a perfect square. (The quantity added must be compensated for in such a way that the new expression or statement is equivalent to the old one, for example by adding the same quantity to both sides of an equation.)

Complex numbers A *complex number* is a symbolic expression involving (real) numbers as well as the imaginary unit i. Symbolically, a complex number has the form $a + bi$, where a and b are (real) numbers.

Conditional statement An algebraic statement is *conditional* if it may be true or false, depending on the values assigned to the variables involved.

Constants *Constants* are numbers, or symbols that represent a definite value.

Contradiction A *contradiction* is an algebraic statement which is false for all values of the variables involved. A contradiction therefore has no solution.

Denominator The *denominator* of a fraction is the number written below the bar, and represents the divisor of the quotient.

Descending order A polynomial is written in *descending order* if terms having higher degree are written to the left of terms having lower degree.

Difference of squares A *difference of squares* is a binomial involving the difference of two terms which are each perfect squares. Symbolically, a difference of squares has the form $a^2 - b^2$, where a and b represent any algebraic expression.

Discriminant The *discriminant* of a quadratic formula of the form $ax^2 + bx + c = 0$ is the quantity $b^2 - 4ac$. The discriminant allows one to determine if the solutions of the given equation are rational or irrational, real or complex, and how many distinct solutions the equation has.

Equivalent fractions Two fractions are *equivalent* if they represent the same quotient.

Fraction in reduced form A fraction is written in *reduced form* if the numerator and denominator are integers that have no factor in common (other than 1).

Integer An *integer* is either a whole number or the opposite of a whole number. The set of all integers can be expressed as
$\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

Degree (of a polynomial) The *degree* of a polynomial is the highest degree of any of its terms.

Degree (of a term) The *degree* of a term in a polynomial in one variable is the exponent of the variable part.

Distributive law The *distributive law* expresses a relationship between the operations of addition and multiplication. It states that the product of a quantity with a sum is the same as the sum of the product of the quantity with each of the summands. It is usually expressed symbolically as $a \cdot (b + c) = a \cdot b + a \cdot c$.

Equation An *equation* is an algebraic statement involving the relation of equality (=). An equation is true if both sides of the statement have the same value.

Equivalent equations Two equations are *equivalent* if they have the same solutions.

Evaluate To *evaluate* an expression means to find its value, generally after performing all indicated operations.

Factor (verb) To *factor* an expression means to write it as a product of two or more factors (usually different than 1).

Factor (of an integer) A *factor* of an integer is another integer which, when multiplied by a third integer, is equal to the original integer. (This is sometimes stated as, "A number which divides the original number evenly.")

Factor (of a polynomial) A *factor* of a polynomial is another polynomial which, when multiplied by a third polynomial, gives the original polynomial.

Factor completely To factor a polynomial completely means to write it as a product of two or more factors, none of which can be factored and further.

Fraction A *fraction* is a symbolic way of writing a quotient that involves two numbers separated by a bar representing the operation of division.

Function notation *Function notation* gives a symbolic way of referring to algebraic expressions. It involves naming the expression with a single letter (like f, g, or P), and indicating the variables on which the expression depends. The notation $f(x)$ (read "f of x") is meant to indicate the expression named f with variable x; $f(3)$ indicates the value of $f(x)$ when x is assigned the value 3.

Graph To *graph* an algebraic statement means to plot all solutions, either on a number line (statements involving one variable) or on an xy-plane (statements involving two variables).

Greatest common divisor The *greatest common factor* (GCF) of monomials (with integer coefficients) is the product of the GCF of the coefficients with the lowest power of each variable appearing in any of the terms.

Horizontal lines A line in an xy-plane is called *horizontal* if it is parallel to the x-axis. Horizontal lines have slope 0.

Hypotenuse In a right triangle, the *hypotenuse* is the side opposite to the right angle. Due to the Pythagorean theorem, it is also the longest side of a right triangle.

Identity An *identity* is an algebraic statement which is true for all values of the variables involved.

Imaginary unit The *imaginary unit*, denoted i, is the symbol defined to have the property that $i^2 = -1$.

Inequality An *inequality* is an algebraic statement involving the relations of "greater than" ($>$) or "less than" ($<$). An inequality is true when the inequality symbol "points" in the direction of the side with the smaller value. Inequalities include the compound inequalities "greater than or equal to" ($\geq$) and "less than or equal to" ($\leq$), which are true when either the inequality is true or when the sides have the same value.

Intercepts (x– and y–intercept of a line in an xy-plane) The *intercepts* of a line drawn in an xy-plane are the points where the line intersects the x-axis (the x-intercept) and where the line intersects the y-axis (the y-intercept). The ordered pair corresponding to the x-intercept will have 0 as a y-coordinate, while the ordered pair corresponding to the y-intercept will have 0 as an x-coordinate.

Irrational numbers A real number is *irrational* if it is impossible to represent it as a ratio of two integers. The decimal expansion of an irrational number never terminates and never repeats.

Leading term The *leading term* of a polynomial is the term with the highest degree. (Hence, when a polynomial is written in descending order, the leading term is the first term written.)

Like terms *Like terms* have the same variable part. For a linear equation in one variable, two terms are like terms if they either both involve the variable or both do not involve the variable. For a polynomial in one variable, like terms are those having the same degree.

Linear equation An algebraic statement is *linear* if the only operations performed on a variable are addition, subtraction, and multiplication by a constant. As a consequence, every term of a linear equation involves at most one variable, and the highest exponent of any variable is 1. In the language of polynomials, an equation is linear if it only involves polynomial expressions of degree 1.

Literal equation A *literal equation* (or *formula*) is an equation that relates several variables, usually meant to express relationships between measured quantities.

Magnitude The *magnitude* of a number is a non-negative number that represents how "big" a number is, regardless of sign. On a number line, the magnitude of a number is the distance from the point representing the number to the point representing 0.

Monic polynomial A polynomial is called *monic* if the coefficient of the leading term is 1.

Multiplication principle The *multiplication principle* expresses the fact that when the same non-zero quantity is multiplied by (or divided by) each side of an algebraic equation, the resulting equation is equivalent to (has the same solutions as) the original one. The multiplication principle also applies to inequalities when multiplying by *positive* quantities. When multiplying an algebraic inequality by a *negative*, the original inequality is equivalent to the new inequality with the opposite sense of the inequality.

Natural number A *natural number* is a number which occurs in the "natural" operation of counting, and are sometimes called "counting numbers." The set of all natural numbers can be expressed as $\{1, 2, 3, \dots\}$.

Negative number A *negative* number is a number which is less than zero. On a number line, negative numbers are represented by points to the left of the point representing zero.

Non-negative number A *non-negative* number is either positive or zero. Said differently, a non-negative number is a number greater than or equal to zero.

Numerator The *numerator* of a fraction is the number written above the bar, and represents the dividend of the quotient.

Opposite of a number The *opposite* of a number is the number which, when added to the original number, gives zero. The opposite of a number is also called the "additive inverse" of a number. The opposite of a number has the same magnitude, but opposite sign as the original number. Symbolically, the opposite of a number a is $-a$.

Order of operations The *order of operations* is the conventional order in which operations are performed when there is more than one operation involved.

Ordered pair An *ordered pair* is a notation that is well-suited to describe solutions of algebraic statements in two variables. It involves two numbers (called *coordinates*) separated by a comma and enclosed in parentheses, for example $(3, 2)$. The first number is traditionally called the x-coordinate while the second number is traditionally called the y-coordinate.

Parabola The shape of the graph of solutions to equations of the form $y = ax^2 + bx + c$ is called a *parabola*. A parabola has a vertex (or turning point) and an axis of symmetry.

Parallel lines Two lines in a plane are *parallel* if they do not intersect. In an xy-plane, parallel lines have the same slope.

Parentheses (and other grouping symbols) *Parentheses*, along with other symbols like brackets, are used in a mathematical expression to indicate grouping. In the order of operations, grouped operations are performed before other operations.

Perpendicular lines Two lines in a plane are *perpendicular* if they meet at a right angle. In an xy-plane, the product of the slopes of perpendicular lines is -1.

Plotting an ordered pair To *plot* an ordered pair is the process of representing an ordered pair with a point in an xy-plane. The coordinates represent distances to the axes, with the distance measured in a direction corresponding to whether the coordinate is positive or negative.

Plus-or-minus ($\pm$) notation The symbol $\pm k$ denoted the compound expression, "Either k or $-k$." An equation $x = \pm k$ is the same as the compound statement, "Either $x = k$ or $x = -k$.

Point-slope form of a linear equation in two variables An equation derived from the formula defining the slope of a line in an xy-plane, which makes explicit the slope of the line and the coordinates of one point. It is written

$$y - y_0 = m(x - x_0),$$

where m is the slope and (x_0, y_0) represent the (constant) coordinates of one point on the line. The point-slope form of a line is generally used as a "formula" to write the equation describing a line in an xy-plane with some given geometric data.

Polynomial A *polynomial* (in one variable) is an algebraic expression, each of whose terms have the form ax^n where a is a constant coefficient and n is a whole number.

Positive number A *positive* number is a number which is greater than zero. On a number line, positive numbers are represented by points to the right of the point representing zero.

Pythagorean theorem The *Pythagorean theorem* expresses a relationship between the lengths of the sides of a right triangle. Specifically, it states that a triangle with hypotenuse with length h and with the two other sides having lengths a and b is a right triangle if and only if $h^2 = a^2 + b^2$.

Quadratic equation A *quadratic equation* is an equation involving polynomials of degree 2. In particular, a polynomial in one variable x must have an x^2 term (with nonzero coefficient), and no term may have degree higher than 2.

Quadratic formula The *quadratic formula* expresses the solutions to a quadratic equation of the form $ax^2+bx+c=0$ in terms of the coefficients a, b and c. Specifically, it states that the equation $ax^2+bx+c=0$ is equivalent to the (compound) statement

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

Quadratic trinomial A *quadratic trinomial* is a polynomial with three terms with degree two. Symbolically, a quadratic trinomial in one variable x can be written ax^2+bx+c, where a, b and c represent constant coefficients.

Radicand The *radicand* of a radical symbol is the quantity enclosed by the radical sign $\sqrt{}$. Symbolically, the radicand of the symbol $\sqrt{k}$ is k.

Rational number A number is said to be *rational* if can be expressed as a ratio of two integers. The decimal expansion of a rational number either terminates or repeats.

Rationalizing (a denominator) The process of multiplying the numerator and denominator of a fraction by (the same) appropriate quantity so that the resulting denominator does not involve a radical symbol after simplifying is called *rationalizing the denominator.*

Real numbers Any number or numerical expression not involving the imaginary unit i is called a *real number.*

Reciprocal The *reciprocal* of a number is the number which, when multiplied by the original number, has product 1. The reciprocal is also called the "multiplicative inverse" or just "inverse." Symbolically, the reciprocal of a number a is $1/a$.

Scientific notation A number is written in *scientific notation* if it has the form $a \times 10^n$, where a is a number whose magnitude is greater than or equal to 1, but strictly less than 10, and where n is an integer.

Slope of a line The *slope* of a line is a number measuring the steepness of the line. In an xy-plane, it is given by the ratio of the change in the y-coordinates of any two points on the line to the change in the x-coordinates of the same two points.

Slope-intercept form of a linear equation in two variables A linear equation in two variables x and y is said to be written in *slope-intercept* *form* if the y variable is by itself on one side of the equation. An equation in slope-intercept form is often written $y = mx + b$, since the coefficient of the variable x represents the slope of the line obtained by plotting solutions to the equation, and b represents the y-coordinate of the y-intercept.

Solution A *solution* to a conditional statement is a value for each variable in the statement which, when substituted into the expressions involved, yield a true statement.

Solve To *solve* a (conditional) algebraic statement means to find all solutions of the statement.

Strict inequality An inequality is *strict* if it involves only the relations of $<$ or $>$, and not the compound inequalities $\leq$ and $\geq$.

System of linear equations A *system* of linear equations consists of two or more equations which are taken as part of a compound statement, usually indicated by writing a brace symbol ($\{$). A solution to a system must be a solution to all of the equations involved in the system.

Term A *term* is an algebraic expression which is not itself the sum of two or more expressions. A general algebraic expression can be written as a sum of terms.

Variables *Variables* are mathematical symbols, usually indicated by letters, that indicate an unknown number, or a number that changes with time.

Vertical lines A line in an xy-plane is called *vertical* if it is parallel to the y-axis. Vertical lines do not have a slope (or the slope is *undefined*).

Whole number A *whole number* is either a natural number or 0. The set of all whole numbers can be expressed as $\{0, 1, 2, 3, \dots\}$. Whole numbers answer the question, "How many?"

xy**-plane** An xy-plane is a method for representing ordered pairs as points in a plane. It involves two fixed perpendicular lines (called *axes*) which intersect at one point (called the *origin*).

Zero product property The *zero product property* of numbers expresses the fact that for any two numbers a and b, $a \cdot b = 0$ implies that either a or b is 0.