

MATH 42 - Linear Algebra

Test 1. Time allowed: one hour. Professor Luis Fernández

NAME: _____

INSTRUCTIONS: Solve the following exercises. **You must show all your work** in order to receive credit.

[12] 1. Determine if the following statements are true or false (circle **T** or **F**). You do not need to justify your answer.

a) **T** **F** If A is an invertible $n \times n$ matrix, then the system $A\vec{x} = \vec{0}$ has infinitely many solutions.

b) **T** **F** If the reduced row echelon form of an $n \times n$ matrix A is I_n , then A is invertible.

c) **T** **F** If A is invertible, then the only solution of the system $A\vec{x} = \vec{b}$ is $\vec{x} = A^{-1}\vec{b}$.

d) **T** **F** If A and B are invertible $n \times n$ matrices, then AB is also invertible.

[22] 2. Solve the system of equations
$$\begin{cases} x - y + 4z = 1 \\ 3x - 2y - 5z = 3 \end{cases}$$
 Express your answer in parametric form.

[22] **3.** Find the value(s) of k so that the following linear system is consistent (that is, has at least one solution):

$$\begin{cases} 3x_1 - 5x_2 = 4 \\ 9x_1 + kx_2 = -1 \end{cases}$$

Find the solution when $k = -2$.

[22] **4.** Find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ 2 & 5 & -4 \\ -1 & -3 & 3 \end{bmatrix}$

[22] **5.** For the matrix $A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}$, find

a) $A^T A$.

b) AA^T .

c) $(A^T A)^{-1}$.

d) [BONUS, +5pt] Show that AA^T is not invertible.