QUIZ 1. SOLUTION.

1

$$\begin{cases} 3x_1 - 5x_2 = 4 \\ 9x_1 + k x_2 = -1 \end{cases}$$ Solve the system!

$$\left(\begin{array}{cc|c} 3 & -5 & 4 \\ 9 & k & -1 \end{array}\right) \to \left(\begin{array}{cc|c} 3 & -5 & 4 \\ 0 & k+15 & 13 \end{array}\right).$$

If $k \neq 15 \to$ system is consistent.

$k = 15 \Rightarrow$ system is inconsistent (no solution).

---

2 We need to solve: $x_1\vec{a}_1 + x_2\vec{a}_2 = \vec{b}$.

Write augmented matrix:

$$\left(\begin{array}{cc|c} -1 & 2 & -10 \\ 4 & 8 & -8 \\ -3 & -7 & 9 \end{array}\right) \to \left(\begin{array}{cc|c} -1 & 2 & -10 \\ 0 & 16 & -48 \\ 0 & -13 & 39 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -2 & 10 \\ 0 & 1 & -3 \\ 0 & 1 & -3 \end{array}\right)$$

$$\to \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right) \Rightarrow x_1 = 4,\ x_2 = -3.$$

Yes, $\vec{b}$ is in the span of $\vec{a}_1, \vec{a}_2$. In fact, $4\vec{a}_1 - 3\vec{a}_2 = \vec{b}$.

---

3 We need to see whether the system $x\vec{u} + y\vec{v} + z\vec{w} = \vec{0}$ has a unique solution.

Write matrix with columns $[\vec{u}\ \vec{v}\ \vec{w}]$:

(OVER)

(3 cont)

$$\begin{pmatrix} 1 & 0 & 2 \\ -1 & 4 & 6 \\ 2 & 1 & 7 \end{pmatrix} \xrightarrow[R_3-2R_1\to R_3]{R_2+R_1\to R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 8 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{\frac{R_2}{4}\to R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 3 \end{pmatrix} \xrightarrow{R_3-R_2} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

Every column has a pivot.
Therefore the solution of $x\vec{u}+y\vec{v}+z\vec{w}=\vec{0}$ is unique $(=(0,0,0))$, so the vectors are linearly independent.

---

④ $T(4\vec{u}_1-3\vec{u}_2)=4T(\vec{u}_1)-3T(\vec{u}_2)$

$$=4\begin{pmatrix}3\\-1\end{pmatrix}-3\begin{pmatrix}4\\5\end{pmatrix}=\begin{pmatrix}12-12\\-4-15\end{pmatrix}=\begin{pmatrix}0\\-19\end{pmatrix}.$$

---

⑤ If $T(\vec{x})=A\vec{x}$, and B is the row-echelon form of A, then:

- Every column has a pivot $\Leftrightarrow$ one to one
- " row " " " $\Leftrightarrow$ onto.

In this case,

$$A=\begin{pmatrix}1 & 2 & -1\\ -2 & 5 & 4\end{pmatrix}\to\begin{pmatrix}1 & 2 & -1\\ 0 & 9 & 2\end{pmatrix}$$

Every row has a pivot, so <u>T onto.</u>

<u>Not</u> every column has a pivot, so T is <u>NOT one to one</u>.

6 $$\begin{pmatrix} 4 & 1 \\ -3 & 0 \\ 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 7 & 3 \\ -3 & -6 & 0 \\ 3 & 1 & 15 \end{pmatrix}$$

7

a) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$,

so $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ is idempotent.

b) Suppose $A^2 = A$.

$$\begin{aligned} (I-A)^2 &= (I-A)(I-A) \\ &= I - A - A + \boxed{A^2} \quad = A \\ &= I - A - \cancel{A} + \cancel{A} \\ &= I - A. \end{aligned}$$

Since $(I-A)^2 = (I-A)$,

$I-A$ is also idempotent.