MATH 42 - Linear Algebra

QUIZ 2. Time allowed: one hour. Professor Luis Fernández

NAME:____

INSTRUCTIONS: Solve the following exercises. **You must show all your work** in order to receive any credit.

[20] **1.** Find the matrix of the change of coordinates from the basis $\mathcal{B}$ to the basis $\mathcal{C}$, where

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix} \right\} \quad \text{and} \quad \mathcal{C} = \left\{ \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \right\}$$

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[20] **2.** Consider the subspace S of $\mathbb{R}^4$ spanned by the elements of the basis $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 4 \\ -2 \end{pmatrix} \right\}$.

Find the vector $\vec{v}$ whose coordinates in the basis $\mathcal{B}$ are $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ (that is, if $[\vec{v}]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, find $\vec{v}$).

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[20] **3.** Find the coordinates of the vector $\vec{v} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$ in the basis $\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}$.

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[20] **4.** Find $\begin{vmatrix} 1 & 0 & 0 & 1 \\ 2 & 1 & 3 & -1 \\ -1 & 0 & -2 & 3 \\ 0 & 1 & 3 & 1 \end{vmatrix}$

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[10] **5.** Suppose that the range of a transformation $T : \mathbb{R}^7 \to \mathbb{R}^3$ has dimension 3. What is the dimension of the kernel of T?

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[10] **6.** Show that $\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = (x_3 - x_2)(x_2 - x_1)(x_3 - x_1)$.

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[10] **7.** (BONUS) Show that

$$\begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 & x_1^4 & x_1^5 \\ 1 & x_2 & x_2^2 & x_2^3 & x_2^4 & x_2^5 \\ 1 & x_3 & x_3^2 & x_3^3 & x_3^4 & x_3^5 \\ 1 & x_4 & x_4^2 & x_4^3 & x_4^4 & x_4^5 \\ 1 & x_5 & x_5^2 & x_5^3 & x_5^4 & x_5^5 \end{vmatrix}$$

$$= (x_5 - x_4)(x_5 - x_3)(x_5 - x_2)(x_5 - x_1)(x_4 - x_3)(x_4 - x_2)(x_4 - x_1)(x_3 - x_2)(x_3 - x_1)(x_2 - x_1).$$

[HINT: use column operations to make zeros in the top row starting with the last column by subtracting from each column a multiple of the previous columns.]