

MATH 42 - Linear Algebra

QUIZ 1. Time allowed: one hour. Professor Luis Fernández

NAME: _____

INSTRUCTIONS: Solve the following exercises. **You must show all your work** in order to receive any credit.

- [15] **1.** Find the value(s) of k so that the following linear system is consistent (that is, has at least one solution):

$$\begin{cases} 3x_1 - 5x_2 = 4 \\ 9x_1 + kx_2 = -1 \end{cases}$$

- [20] **2.** Determine if the vector $\vec{b} = \begin{pmatrix} -10 \\ -8 \\ 9 \end{pmatrix}$ is in the span of the vectors $\vec{a}_1 = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $\vec{a}_2 = \begin{pmatrix} 2 \\ 8 \\ -7 \end{pmatrix}$.
If it is, write \vec{b} as a linear combination of \vec{a}_1 and \vec{a}_2 .
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- [15] **3.** Determine if the vectors $\vec{u} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $\vec{v} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$, $\vec{w} = \begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}$ are linearly independent.
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- [15] **4.** Suppose that a linear transformation $T : \mathbb{R}^n \rightarrow \mathbb{R}^2$ satisfies $T(\vec{u}_1) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $T(\vec{u}_2) = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$.
Find $T(4\vec{u}_1 - 3\vec{u}_2)$.
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- [20] **5.** Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be the linear transformation given by $T(\vec{x}) = \begin{pmatrix} 1 & 2 & -1 \\ -2 & 5 & 4 \end{pmatrix} \vec{x}$.
a) Determine if T is one to one.
b) Determine if T is onto.
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- [15] **6.** Multiply the following matrices:

$$\begin{pmatrix} 4 & 1 \\ -3 & 0 \\ 3 & 5 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 0 \\ 0 & -1 & 3 \end{pmatrix}$$

- [10] **7.** BONUS. A matrix A is called *idempotent* if $A^2 = A$. For example, I and $0_{2 \times 2}$ are idempotent.
a) Find a 2×2 matrix, not equal to I or $0_{2 \times 2}$, that is idempotent.
b) Prove that if A is idempotent, then so is $I - A$.