MATH 34 – Differential equations.

Second in-class test. Time allowed: two hours.

Instructor: Luis Fernández

NAME:____

Directions: Answer the following questions. The exam will be graded over 100 points; any points you get over 100, up to 110, will count as extra credit. The number to the left of each question is the number of points it is worth.

[20] **1.** Answer the following short questions, justifying your answer:

a) Suppose that, $y_H(t)$ is a solution of the differential equation $\frac{dy}{dt} = a(t)y$, and $y_P(t)$ is a solution of the differential equation $\frac{dy}{dt} = a(t)y + b(t)$. Write down **three more solutions** of the differential equation $\frac{dy}{dt} = a(t)y + b(t)$.

b) Determine if the following statement is true or false, justifying your answer: 'A linear system of differential equations always has $x(t) = 0$, $y(t) = 0$ as an equilibrium solution'.

c) Write down a differential equation that has $y(t) = e^{2t}$ as a solution.

d) Convert the second order linear differential equation $\dfrac{d^2y}{dt^2} + 4\dfrac{dy}{dt} - 3y = 0$ into a system of differential equations.

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[12] **2.** Find the general solution of the differential equation

$$\frac{dy}{dt} - \frac{3}{t}y = 2t^3 \cos 2t.$$

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[12] **3.** Solve the initial-value problem $\dfrac{dy}{dt} = -5y + 3e^t, \quad y(0) = 2.$

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[12] **4.** Find the general solution of the (partially decoupled) system $\dfrac{d\vec{Y}}{dt} = \begin{pmatrix} 2 & 0 \\ 2 & -1 \end{pmatrix} \vec{Y}$

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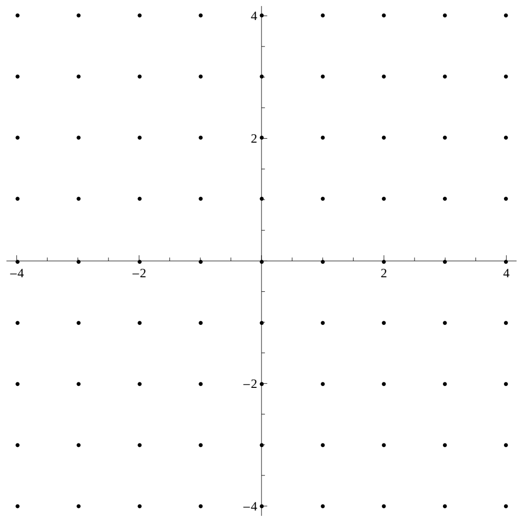
[12] **5.** Find all the equilibrium solutions for the system

$$\begin{cases} \dfrac{dx}{dt} &= x(2x + y - 6) \\ \dfrac{dy}{dt} &= y(-2x + y - 2) \end{cases}$$

[12] **6.** Draw, in the axes provided below, the vector field and phase portrait for the system

$$\begin{cases} \dfrac{dx}{dt} & = & 1 \\ \dfrac{dy}{dt} & = & 0 \end{cases}$$



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[12] **7.** For the initial value problem

$$\begin{cases} \dfrac{dx}{dt} & = & -y \\ \dfrac{dy}{dt} & = & 2x - y \\ x(0) & = & 1 \\ y(0) & = & 1 \end{cases}$$

use Euler's method with step size $\Delta t = 1$ to approximate $\vec{Y}(4)$.

[NOTE: you can use a calculator here.]

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[12] **8.** Let $x(t)$ and $y(t)$ be the populations (at time t) of two species of animals. The behavior of the two populations is modelled by the system of equations

$$\begin{cases} \dfrac{dx}{dt} & = & 10x - xy \\ \dfrac{dy}{dt} & = & 4y - 2xy \end{cases}$$

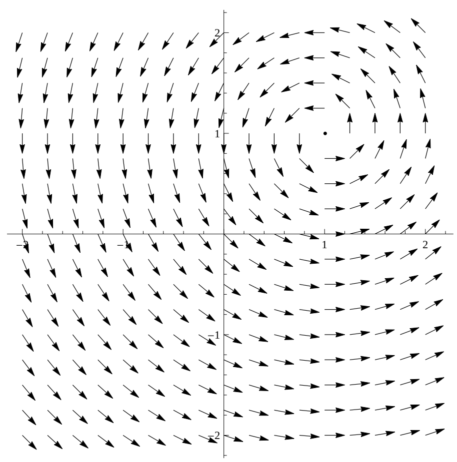
a) What happens to each species in the absence of the other?
b) How do the two species get along? How do you know?
c) What are the equilibrium population levels?

[12] **9.** Eight systems of equation and four direction fields are given below. Determine the system that corresponds to each direction field and state briefly how you know that your choice is correct.

a) $\begin{cases} \dfrac{dx}{dt} &= x \\ \dfrac{dy}{dt} &= y \end{cases}$ **b)** $\begin{cases} \dfrac{dx}{dt} &= -y \\ \dfrac{dy}{dt} &= x \end{cases}$ **c)** $\begin{cases} \dfrac{dx}{dt} &= -y+1 \\ \dfrac{dy}{dt} &= x-1 \end{cases}$

d) $\begin{cases} \dfrac{dx}{dt} &= \cos(\pi x) \\ \dfrac{dy}{dt} &= 0 \end{cases}$ **e)** $\begin{cases} \dfrac{dx}{dt} &= 1 \\ \dfrac{dy}{dt} &= 3xy \end{cases}$ **f)** $\begin{cases} \dfrac{dx}{dt} &= x-1 \\ \dfrac{dy}{dt} &= y^2-1 \end{cases}$

(i)



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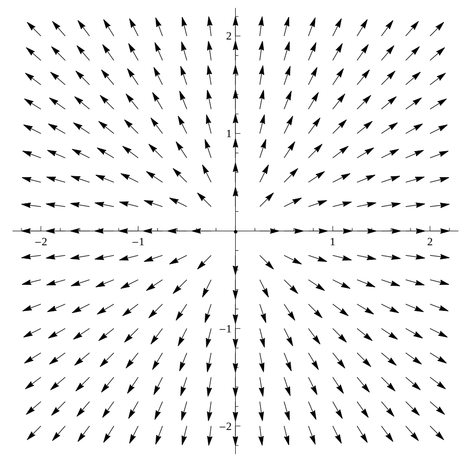

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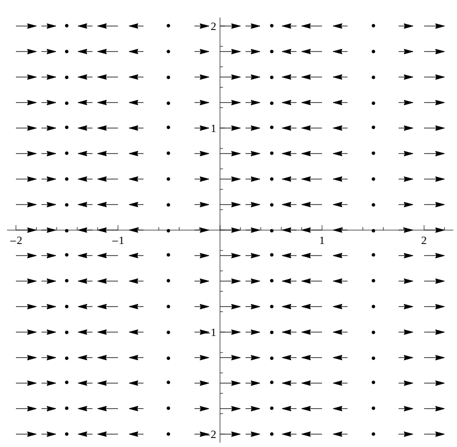

(iv)