MATH 34 – Differential equations.

First in-class test. Time allowed: two hours.

Instructor: Luis Fernández

NAME:____

Directions: Answer at least 5 out of the 7 questions below. The exam will be graded over 100 points, but any points you get over 100 will count as extra credit, up to a maximum of 115 points. Each exercise is worth 20 points.

[20] **1.** Answer the following short questions, justifying your answer:

a) Determine whether the function $y(t) = \frac{1}{t-1}$ is a solution of the equation $\frac{dy}{dt} = -y^2$.

b) Find a solution of the initial value problem $\frac{dy}{dt} = e^y - 1,\ y(0) = 0$. [NOTE: it is easy to guess a solution.]

c) Find one particular solution of the equation $\frac{dy}{dt} + 2y^2 = 8,\quad y(0) = 2$. [NOTE: it is easy to guess a solution.]

d) Determine if the following statement is true or false: 'every autonomous equation is separable'.

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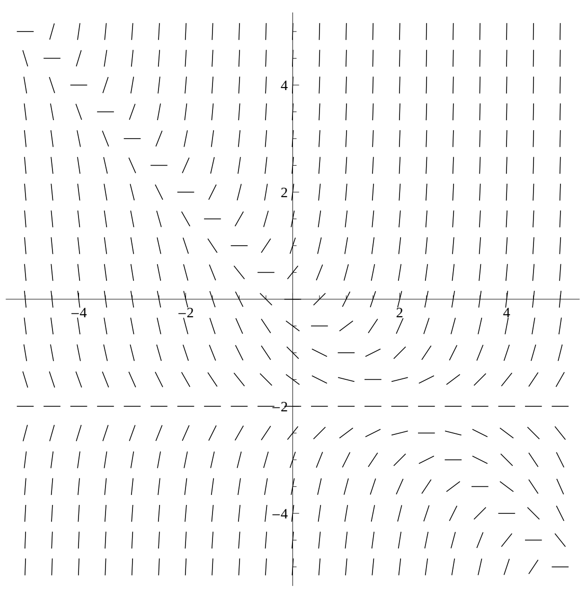
[20] **2.** Which of the differential equations below has the following slope field?



a) $\frac{dy}{dt} = 2y - 1$

b) $\frac{dy}{dt} = (y+2)(y+t)$

c) $\frac{dy}{dt} = t(y^2 - \frac{1}{4})$

d) $\frac{dy}{dt} = \sin t$

e) $\frac{dy}{dt} = t$

f) $\frac{dy}{dt} = y$

[20] **3.** Solve the initial value problem

$$\frac{dy}{dt} = y^2 \cos t + \cos t, \quad y(0) = 0.$$

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[20] **4.** Use Euler's method with step size $\Delta t = 1$ to approximate the solution $y(t)$ at $t = 3$ of the initial value problem

$$\frac{dy}{dt} = t^2 y + 2, \quad y(0) = 0.$$

[NOTE: you can use a calculator here.]

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[20] **5.** The air in a small room 30 ft by 5 ft by 10 ft is 5% carbon monoxide. At time 0, air containing 2% carbon monoxide is blown into the room at the rate of 20 ft^3 per minute and well mixed air flows out of the room at the same rate.

a) Write down an initial value problem for the amount of carbon monoxide in the room over time.

b) How much carbon monoxide will be in the room after a very long time?

c) How much carbon monoxide will be in the room after 10 minutes?

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[20] **6.** Consider the autonomous differential equation $\dfrac{dy}{dt} = (y+1)^2(y-1)(y-6)^3(y-10)^2.$

a) Sketch the phase line for this equation and classify the equilibrium points (sink, source or node).

b) Sketch the graph, in the ty-plane, of some of the solutions of this equation.

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[20] **7.** Consider the autonomous differential equation $\dfrac{dy}{dt} = f(y)$, where f is a function that is continuously differentiable.

a) Suppose that $y_1(t)$ is a solution of this equation and $y_1(t)$ has a maximum at $t = t_0$. Let $y_0 = y(t_0)$. Show that $f(y_0) = 0$.

b) Show that the constant function $y_2(t) = y_0$ is a solution.

c) Show that $y_1(t) = y_0$.