MTH 32. QUIZ 4. NAME:______________________________

[12] **1.** Find the following limits, justifying your answer:

a) $\lim_{x\to 0} \dfrac{e^{2t}-1}{\sin x}$.

Solution: $\lim_{x\to 0}(e^{2t}-1)=0$ and $\lim_{x\to 0}\sin x = 0$. Therefore the limit has the form $\frac{0}{0}$ and we can use L'Hopital's rule:

$$\lim_{x\to 0}\frac{e^{2t}-1}{\sin x}=\lim_{x\to 0}\frac{2e^{2t}}{\cos x}=\frac{2}{1}=2.$$

b) $\lim_{\theta\to\pi/2}\dfrac{1-\sin\theta}{1+\cos 2\theta}$.

Solution: $\lim_{\theta\to\pi/2}(1-\sin\theta)=0$ and $\lim_{\theta\to\pi/2}(1+\cos 2\theta)=0$. Therefore the limit has the form $\frac{0}{0}$ and we can use L'Hopital's rule:

$$\lim_{\theta\to\pi/2}\frac{1-\sin\theta}{1+\cos 2\theta}=\lim_{\theta\to\pi/2}\frac{-\cos\theta}{-2\sin 2\theta}.$$

Again, $\lim_{\theta\to\pi/2}(-\cos\theta)=0$ and $\lim_{\theta\to\pi/2}(-2\sin 2\theta)=0$, so we use L'Hopital again:

$$\lim_{\theta\to\pi/2}\frac{1-\sin\theta}{1+\cos 2\theta}=\lim_{\theta\to\pi/2}\frac{-\cos\theta}{-2\sin 2\theta}=\lim_{\theta\to\pi/2}\frac{\sin\theta}{-4\cos 2\theta}=\frac{\sin\pi/2}{-4\cos 2\pi/2}=\frac{1}{4}.$$

c) $\lim_{x\to 0}\dfrac{\sqrt{1+2x}-\sqrt{1-4x}}{x}$.

Solution: $\lim_{x\to 0}(\sqrt{1+2x}-\sqrt{1-4x})=0$ and $\lim_{x\to 0}0=0$. Therefore we can use L'Hopital's rule:

$$\lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt{1-4x}}{x}=\lim_{x\to 0}\frac{\frac{2}{2\sqrt{1+2x}}-\frac{-4}{2\sqrt{1-4x}}}{1}=\frac{2}{2\sqrt{1}}-\frac{-4}{2\sqrt{1}}=3.$$

Alternatively, multiply numerator and denominator by the conjugate of the numerator:

$$\begin{aligned}\lim_{x\to 0}\frac{\sqrt{1+2x}-\sqrt{1-4x}}{x}&=\lim_{x\to 0}\frac{(\sqrt{1+2x}-\sqrt{1-4x})(\sqrt{1+2x}+\sqrt{1-4x})}{x(\sqrt{1+2x}+\sqrt{1-4x})}\\&=\lim_{x\to 0}\frac{(1+2x)-(1-4x)}{x(\sqrt{1+2x}+\sqrt{1-4x})}\\&=\lim_{x\to 0}\frac{6x}{x(\sqrt{1+2x}+\sqrt{1-4x})}\\&=\lim_{x\to 0}\frac{6}{(\sqrt{1+2x}+\sqrt{1-4x})}\\&=\frac{6}{2}\\&=3.\end{aligned}$$