

MTH 32. QUIZ 3. NAME: _____

- [4] **1.** Prove the identity $\sinh x + \cosh x = e^x$.

Solution:

$$\begin{aligned}\sinh x + \cosh x &= \frac{e^x - e^{-x}}{2} + \frac{e^x + e^{-x}}{2} \\ &= \frac{e^x - e^{-x} + e^x + e^{-x}}{2} \\ &= \frac{2e^x}{2} \\ &= \boxed{e^x}\end{aligned}$$

- [4] **2.** Find the derivative of the function $f(t) = e^{\cosh 3t}$.

Solution:

$$f'(t) = e^{\cosh 3t} \cdot (\cosh 3t)' = e^{\cosh 3t} \cdot \sinh 3t \cdot 3 = \boxed{3 \cdot e^{\cosh 3t} \cdot \sinh 3t}.$$

- [4] **3.** Find $\lim_{x \rightarrow \infty} \tanh x$.

Solution:

$$\lim_{x \rightarrow \infty} \tanh x = \lim_{x \rightarrow \infty} \frac{\sinh x}{\cosh x} = \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}.$$

Both numerator and denominator tend to ∞ when $x \rightarrow \infty$ due to the e^x . Therefore, divide numerator and denominator by e^x :

$$\lim_{x \rightarrow \infty} \tanh x = \lim_{x \rightarrow \infty} \frac{e^x - e^{-x}}{e^x + e^{-x}} = \lim_{x \rightarrow \infty} \frac{\frac{e^x}{e^x} - \frac{e^{-x}}{e^x}}{\frac{e^x}{e^x} + \frac{e^{-x}}{e^x}} = \lim_{x \rightarrow \infty} \frac{1 - \frac{1}{e^{2x}}}{1 + \frac{1}{e^{2x}}} = \boxed{1},$$

since $\lim_{x \rightarrow \infty} \frac{1}{e^{2x}} = 0$.