MTH 32. QUIZ 2. NAME:______________________________

[4] **1.** Find the exact value of

a) $\csc^{-1}\sqrt{2}$

Solution: Which angle between $-\pi/2$ and $\pi/2$ has a cosecant equal to $\sqrt{2}$? That is: If $\csc^{-1}\sqrt{2} = \theta$, how much is θ?

Well, if $\csc^{-1}\sqrt{2} = \theta$ then $\csc\theta = \sqrt{2}$, and therefore, since $\csc\theta = \dfrac{1}{\sin\theta}$, we must have $\sin\theta = 1/\sqrt{2}$, which implies that $\theta = \pi/4$. Therefore,

$$\csc^{-1}\sqrt{2} = \frac{\pi}{4}.$$

b) $\cos^{-1}(\sqrt{3}/2)$

Solution: If $\theta = \cos^{-1}(\sqrt{3}/2)$, then $\cos\theta = \sqrt{3}/2$ and θ is between 0 and π, which implies that $\theta = \pi/6$. Therefore,

$$\cos^{-1}(\sqrt{3}/2) = \frac{\pi}{6}.$$

[4] **2.** Find the derivative of the function $f(x) = x\sin^{-1}x + \sqrt{1-x^2}$.

Solution: Recall that $(\sin^{-1}x)' = \dfrac{1}{\sqrt{1-x^2}}$. Therefore

$$f'(x) = \sin^{-1}x + x\frac{1}{\sqrt{1-x^2}} + \frac{-2x}{2\sqrt{1-x^2}} = \sin^{-1}x + \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}} = \sin^{-1}x,$$

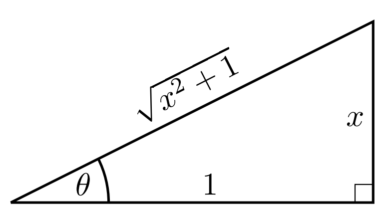
so

$$f'(x) = \sin^{-1}x.$$

[4] **3.** Simplify the expression $\sin(\tan^{-1}x)$.

Solution: Break it down: write $\theta = \tan^{-1}x$. We want to find $\sin\theta$.

If $\theta = \tan^{-1}x$ then $\tan\theta = x = \frac{x}{1}$. Remember that tan is the opposite side divided by the adjacent side. Do a picture of a right triangle with angle θ, side opposite to θ equal x and side adjacent to θ equal 1. Then the hypotenuse is $\sqrt{x^2+1}$, which implies that $\sin\theta = x/\sqrt{x^2+1}$.



Therefore

$$\sin(\tan^{-1}x) = \frac{x}{\sqrt{x^2+1}}.$$