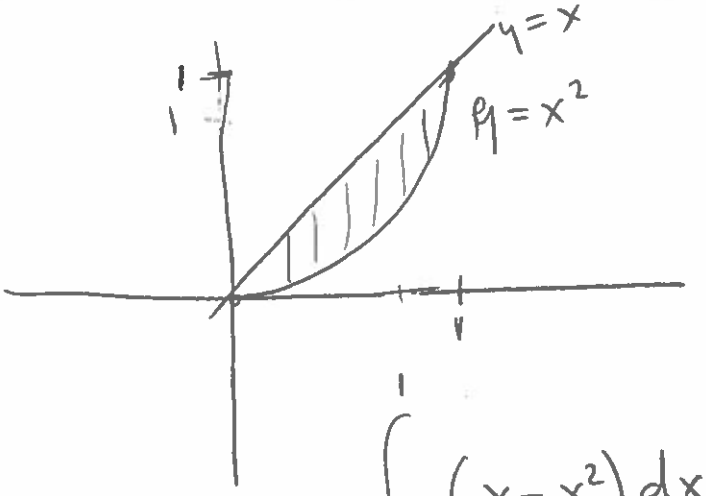
MTH 32. QUIZ 1. NAME: KEY

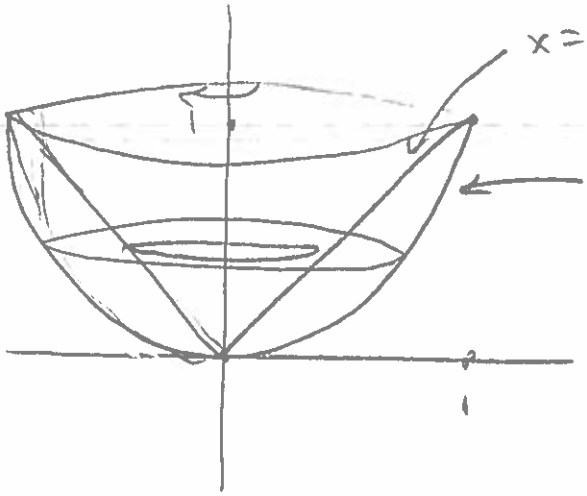
[10] 1. Find the area of the region bounded by the graphs of the functions $f(x) = x^2$ and $g(x) = x$.



$$\int_0^1 (x - x^2)\,dx = \frac{x^2}{2} - \frac{x^3}{3}\Big]_0^1$$

$$= \frac{1}{2} - \frac{1}{3} = \boxed{\frac{1}{6}}$$

[10] **2.** Find the volume of the solid of revolution obtained by rotating the region bounded by the graphs of the functions $f(x) = x^2$ and $g(x) = x$ about the y-axis..



Using washers:

Integrate w.r. to y.

Limits: 0 to 1.

Outside radius: $\sqrt{y}$

inside radius: y

$$\pi \int_0^1 \left((\sqrt{y})^2 - y^2\right) dy = \pi \int_0^1 (y - y^2)\, dy = \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big]_0^1$$

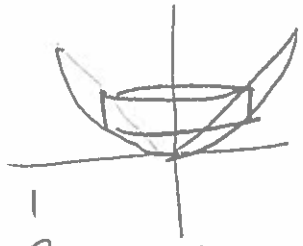
$$= \pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi}{6}.$$

Using shells:

Integrate w.r. to x.

height: from x^2 to x

$\Rightarrow$ height $= x - x^2$



$$\Rightarrow \quad 2\pi \int_0^1 x(x - x^2)\, dx = 2\pi \int_0^1 (x^2 - x^3)\, dx = 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4} \right) \Big]_0^1$$

$$= 2\pi \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{2\pi}{12} = \frac{\pi}{6}.$$