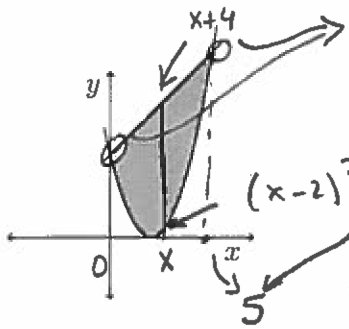
**Print Name:** SOLUTION

**Spring 2018 MTH 32 Test 1**

> **Directions:** Write your answers in the provided space. **Show all work.**

1. (10 pts) Find the area of the region bounded by $y = x + 4$ and $y = (x - 2)^2$:

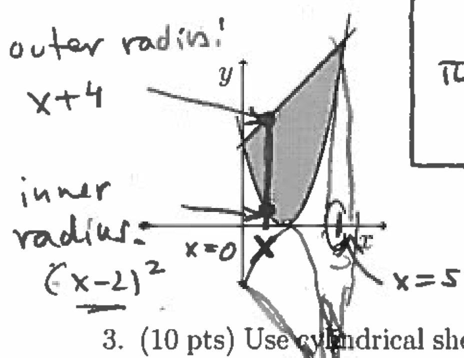



$$x+4=(x-2)^2 \Rightarrow x+\cancel{4} = x^2-4x+\cancel{4} \Rightarrow x^2-5x=0 \Rightarrow x=0, 5$$

$$\int_0^5 \left((x+4)-(x-2)^2\right)dx = \frac{(x+4)^2}{2}-\frac{(x-2)^3}{3}\Big]_0^5$$

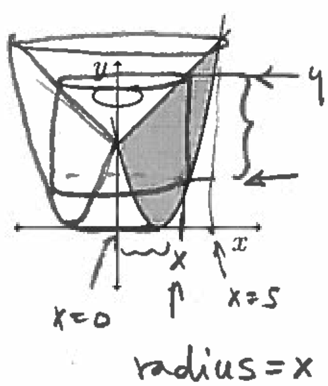
$$= \left(\frac{9^2}{2}-\frac{3^3}{3}\right)-\left(\frac{4^2}{2}-\frac{(-2)^3}{3}\right) = \frac{81-16}{2} - \frac{27+8}{3} = \boxed{\frac{125}{6}}$$

2. (10 pts) Use the washer method to find the volume of the solid obtained by rotating the region bounded by $y = x + 4$ and $y = (x - 2)^2$ around the $x$-axis. Set up the integral only. DO NOT evaluate the integral.




$$\boxed{\text{VOLUME} = \pi\int_0^5 \left((x+4)^2-((x-2)^2)^2\right)dx}$$

3. (10 pts) Use cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by $y = x + 4$ and $y = (x - 2)^2$ around $y$-axis. Set up the integral only. DO NOT evaluate the integral.



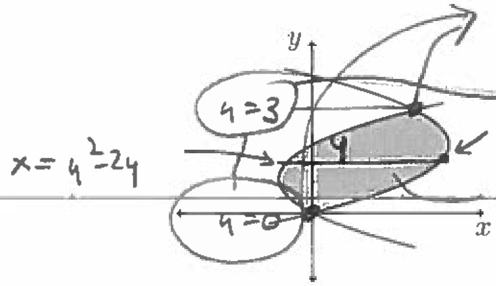

$y=x+4$

$y=(x-2)^2$ $\Big\}$ height $= (x+4)-(x-2)^2$

$$\boxed{\text{Volume} = 2\pi\int_0^5 x\left[(x+4)-(x-2)^2\right]dx}$$

4. (10 pts) Find the area of the region bounded by $x = y^2 - 2y$ and $x = 4y - y^2$. Set up the integral only. DO NOT evaluate the integral.

$y^2 - 2y = 4y - y^2 \Rightarrow 2y^2 - 6y = 0 \Rightarrow 2y(y-3) = 0$
$\Rightarrow y = 0, 3.$




distance: $(4y-y^2) - (y^2-2y)$

$$\boxed{\text{Area} = \int_0^3 \left((4y-y^2) - (y^2-2y)\right) dy}$$

5. (10 pts) Let $f(x) = x^5 + x + 3$. Show that $f$ has an inverse function $g(x)$ (you need to make a clear and convincing argument, using the results from class). Then find $g'(\cancel{21})$ 37.
[Hint: First show that $f(x)$ is monotone. Then: how much is $f(2)$?]

$f'(x) = 5x^4 + 1 \geq 1 > 0$ (because $5x^4 \geq 0$, so $5x^4 + 1 \geq 1$).
Therefore, $f$ is strictly increasing. This implies that $f$ is one to one, which implies that $f$ has an inverse.

$g'(37) = \frac{1}{f'(g(37))}$. Now, $f(2) = 37$, so $g(37) = 2$. $f'(2) = 5 \cdot 2^4 + 1 = 81$

Thus, $\boxed{g'(21) = \frac{1}{81}}$

6. (18 pts) Differentiate the following functions. Do not simplify.

(a) $f(x) = x^e + e^x$.

$$\boxed{f'(x) = e x^{e-1} + e^x}$$

(b) $f(x) = x\,(\ln x)^3$

$$\boxed{f'(x) = (\ln x)^3 + x \cdot 3(\ln x)^2 \cdot \frac{1}{x}}$$

(c) $f(x) = \cos(e^{-x} + 1)$.

$$\boxed{f'(x) = -\sin(e^{-x}+1) \cdot e^{-x} \cdot (-1)}$$

(d) $f(x) = e^x \tan(3x + 2)$

$$\boxed{f'(x) = e^x \cdot \tan(3x+2) + e^x \cdot \sec^2(3x+2) \cdot 3}$$

(e) $f(x) = 2^{\sqrt{x}}$.

$$\boxed{f'(x) = 2^{\sqrt{x}} \cdot \ln 2 \cdot \frac{1}{2\sqrt{x}}}$$

(f) $f(x) = \log_4 \sqrt[5]{x}$

$$\boxed{f'(x) = \frac{1}{\sqrt[5]{x} \cdot \ln 4} \cdot \frac{1}{5} x^{-4/5}}$$

$(x^{1/5})' = \frac{1}{5} x^{-4/5}$

7. (10 pts) Use logarithmic differentiation to find the derivative of the following function. You do not need to simplify.

$$y = f(x) = \frac{x^2\sqrt{x^2+2}}{(x+5)^4}.$$

$$\ln y = 2\ln x + \frac{1}{2}\ln(x^2+2) - 4\ln(x+5).$$

$$\frac{1}{y}y' = \frac{2}{x} + \frac{\cancel{2}x}{2(x^2+2)} - \frac{4}{x+5}$$

$$\Rightarrow \boxed{y' = \frac{x^2\sqrt{x^2+2}}{(x+5)^4}\left(\frac{2}{x} + \frac{\cancel{2}x}{2(x^2+2)} - \frac{4}{x+5}\right)}$$

8. (10 pts) Find the following integrals:

(a) $\int xe^{x^2}\,dx$

Write $u = x^2$, $du = 2x\,dx$

$e^u$

$\frac{du}{2}$

$$\int x e^{x^2} dx = \frac{1}{2}\int e^u du$$

$$= \frac{1}{2}e^u + C$$

$$= \boxed{\frac{1}{2}e^{x^2} + C}$$

(b) $\int \frac{1}{4x-1}\,dx$

put $u = 4x-1$

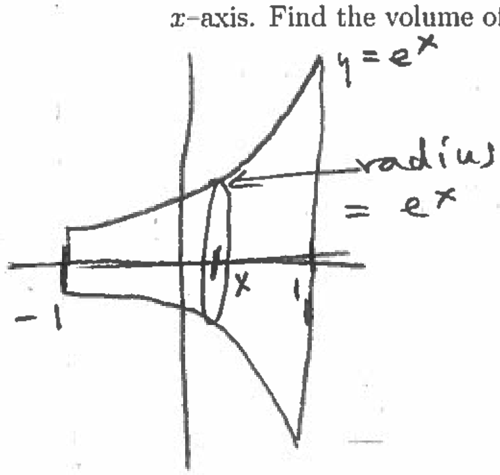
$du = 4dx$

$\frac{1}{u}$ $\quad \frac{du}{4}$

$$= \frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln|u| + C$$

$$= \boxed{\frac{1}{4}\ln|4x-1| + C}$$

9. (12 pts) The region bounded by $y = e^x$, $y = 0$, $x = -1$ and $x = 1$ is revolved about the $x$–axis. Find the volume of the resulting solid.




$$\pi\int_{-1}^{1}(e^x)^2 dx = \pi\int_{-1}^{1} e^{2x} dx$$

$$= \pi \left.\frac{e^{2x}}{2}\right]_{-1}^{1} = \boxed{\frac{\pi}{2}\left(e^2 - e^{-2}\right)}$$

**Bonus:**

1. (5 pts) Find the limit:

$$\lim_{x\to 0^+} x^{\frac{1}{x}} = \lim_{x\to 0^+} e^{\ln(x^{1/x})}$$

$$= \lim_{x\to 0^+} e^{\frac{1}{x}\ln x} = \lim_{u\to -\infty} e^u = \boxed{0}$$

call this $u$.

As $x \to 0^+$, $\frac{1}{x}\ln x \to -\infty$

$\ln x \to -\infty$, $\frac{1}{x} \to \infty$

2. (5 pts) Given $x^y = y^x$. Find $\frac{dy}{dx}$

Take $\ln$ of both sides:

$y \cdot \ln x = x \cdot \ln y$. Take $\frac{d}{dx}$ of both sides!

$$\Rightarrow \frac{dy}{dx} \cdot \ln x + \frac{y}{x} = \ln y + \frac{x}{y} \frac{dy}{dx}$$

Solve for $\frac{dy}{dx}$!

$$\frac{dy}{dx}\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{y\ln x - x}{y}\right) = \frac{x\ln y - y}{x} \Rightarrow \boxed{\frac{dy}{dx} = \frac{(x\ln y - y)y}{(y\ln x - x)x}}$$

110 points total