Spring 2018 MTH 32. Review sheet for test 2

1. Find the exact value of

   (a) $\sin^{-1} \dfrac{\sqrt{2}}{2}$

   (b) $\cos^{-1} \dfrac{\sqrt{2}}{2}$

   (c) $\tan^{-1} \dfrac{1}{\sqrt{3}}$

   (d) $\sin^{-1} \left(\sin \dfrac{3\pi}{2} \right)$

   (e) $\cos^{-1} \left(\cos \dfrac{11\pi}{6} \right)$

   (f) $\sinh 0$

   (g) $\tanh(\mathrm{Ln}\, 4)$

   (h) $\sinh^{-1} 1$

2. Simplify the expressions

   (a) $\tan \left(\sin^{-1} x \right)$

   (b) $\cos \left(\tan^{-1} x \right)$

   (c) $\sinh(\cosh^{-1} x)$

   (d) $\tanh(\cosh^{-1} x)$

   (e) $\sin^{-1} \left(\cos x \right)$ [What is $\sin^{-1} x + \cos^{-1} x$?]

3. Find the derivative of the following functions and simplify.

   (a) $\tan^{-1}(x^2 + 1)$

   (b) $\sec^2(x^4 + 1)$

   (c) $x \cos^{-1} x - \sqrt{1 - x^2}$

   (d) $\cos^{-1}(\sin^{-1}(\cos^{-1} x))$

   (e) $\cosh(x^2 + 5)$

   (f) $\sinh(\mathrm{Ln}\, t)$

   (g) $x \tanh \sqrt{x}$

   (h) $e^{\cosh x}$

4. Evaluate the integrals using substitution if necessary.

   (a) $\displaystyle \int \frac{5}{x^2 + 1}\, dx$

   (b) $\displaystyle \int \frac{5}{\sqrt{1 - x^2}}\, dx$

   (c) $\displaystyle \int \frac{5}{\sqrt{1 + x^2}}\, dx$

   (d) $\displaystyle \int_0^{\pi/2} \frac{\sin x}{1 + \cos^2 x}\, dx$

   (e) $\displaystyle \int \tan x \sec^2 x\, dx$

   (f) $\displaystyle \int_0^{\pi/4} \tan^3 x\, dx$

   (g) $\displaystyle \int_0^{\pi/2} \sin^3 x \cos x\, dx$

   (h) $\displaystyle \int_0^{\pi/2} \cos^3 x\, dx$

5. Evaluate the integrals using integration by parts (and substitution if necessary).

   (a) $\displaystyle \int x \sec^2 x\, dx$

   (b) $\displaystyle \int \mathrm{Ln}\, \sqrt{x}\, dx$

   (c) $\displaystyle \int (x^2 + 1) \sin x\, dx$

   (d) $\displaystyle \int x \sec^2 x\, dx$

   (e) $\displaystyle \int \frac{x}{e^x}\, dx$

   (f) $\displaystyle \int_{-1}^{1} x^3 \cosh x\, dx$

(g) $\displaystyle\int_0^1 x^4 e^x \, dx$

(i) $\displaystyle\int x^4 e^x \, dx$

(h) $\displaystyle\int x^4 e^x \, dx$

(j) $\displaystyle\int x^4 e^x \, dx$

6. Prove the identities

(a) $\cosh 2x = \cosh^x + \sinh^2 x$

(b) $\dfrac{1 + \tanh x}{1 - \tanh x} = e^{2x}$

7. Find the following limits

(a) $\displaystyle\lim_{x \to 0} \dfrac{\cosh x - 1}{x^2}$

(f) $\displaystyle\lim_{x \to \pi/2} \dfrac{\sin x + 1}{\sin x - 1}$

(b) $\displaystyle\lim_{x \to \infty} \dfrac{\cosh x}{x^2}$

(g) $\displaystyle\lim_{x \to \infty} x^{e^{-x}}$

(c) $\displaystyle\lim_{x \to -\infty} x^{1/3} e^x$

(h) $\displaystyle\lim_{x \to \infty} x(\tan^{-1} x - 1)$

(d) $\displaystyle\lim_{x \to -1} \dfrac{x \sin(x + 1)}{3x^2 + 4x + 1}$

(i) $\displaystyle\lim_{x \to \pi/2} (\sec x - \tan x)$

(e) $\displaystyle\lim_{x \to 0} \dfrac{\sinh x - x}{x - \sin x}$

(j) $\displaystyle\lim_{x \to 0^+} (\tan 2x)^x$