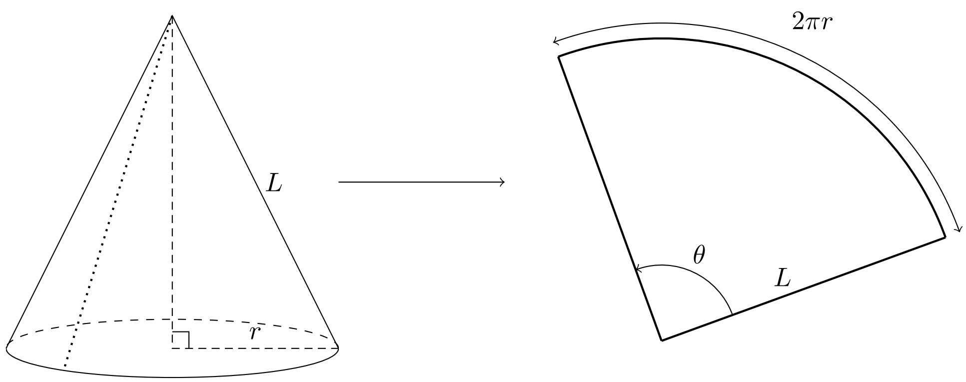
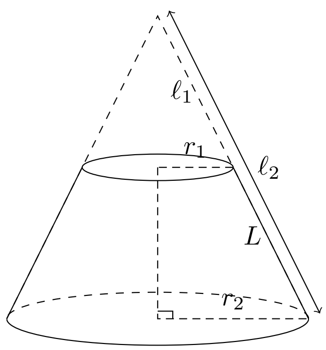
Directions: Do in your own paper. You can work in groups. Hand it in on Thursday.

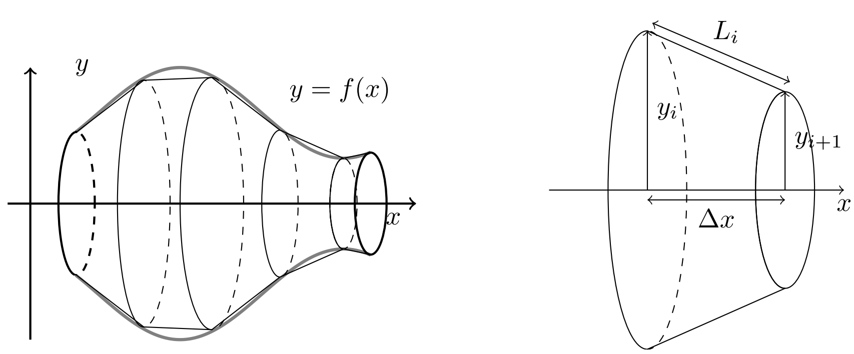
1. Find the surface area of a cone, as in the picture below, as a function of L and r. To do this, cut along the dotted line and open it up to obtain a flat circular sector as in the figure in the left. You need to remember how to find the length of an arc and the area of a sector given the angle θ.



2. Given the previous exercise, show that the area of the truncated cone below is $\pi(r_2 + r_1)L$. To do this first write the area in terms of ℓ_1, ℓ_2, r_1, r_2 and then use similar triangles to write ℓ_1 and ℓ_2 in terms of L, r_1, r_2.



3. Now, given a surface of revolution, given by rotating the graph of $y = f(x)$ around the x-axis, we can slice it so that each piece is approximately a truncated cone, as in the figure below. Each truncated cone looks like the figure on the right.



We know from the last class (when we studied arc length) that

$$L_i = \Delta x \sqrt{1 + (f'(x_i^*))^2}.$$

Also, when Δx is small,

$$(\star) \qquad y_i = f(x_i) \approx f(x_i^*) \text{ and } y_{i+1} = f(x_{i+1}) \approx f(x_i^*).$$

Show that the area of this cone is approximately

$$2\pi f(x_i^*)\sqrt{1 + (f'(x_i^*))^2}\Delta x.$$

Write down the Riemann sum that we get by adding up all the pieces, and conclude that

$$(\star) \qquad \text{Area}(S) = 2\pi \int_a^b f(x)\sqrt{1 + (f'(x))^2}dx.$$

4. Use the previous exercise to find that the area of a sphere of radius R is $4\pi R$.
5. Use the previous exercise to find that the area of the paraboloid obtained by rotating the curve $y = ax^2$ between $x = 0$ and $x = b$ (of course your answer depends on a and b.
6. Use the previous exercise to find that the area of the surface obtained by rotating the curve $y = x^3$ between $x = 0$ and $x = 3$ (of course your answer depends on a and b.
7. Find the area of the infinite surface obtained by rotating the graph of $y = 1/x$, $x \geq 1$, around the x-axis.
8. Find the area of the torus obtained by rotating the circle $x^2 + (y - b)^2 = a^2$ (where $0 < a < b$) around the x-axis. You may need to split it in two parts since when you solve for y in terms of x, you get two different functions.

($\star$) (Of course, we only have proved that the area is approximately equal to the right hand side. To prove that it is actually equal requires a more careful treatment. After you finish the exercises, try to show rigorously that the approximation $f(x_i) \approx f(x^*)$ still gives the right value when $\Delta x \to 0$. The idea is that the error when we approximate $f(x_i)$ with $f(x_i^*)$ is equal to $|f'(\xi)(x_i - x_i^*)|$, where ξ is some number between x_i and x_i^* (using the mean value theorem). This is less than some constant number times Δx, so when we add up these errors in the Riemann sum, we will get a factor of $(\Delta x)^2$, which makes the sum of the errors converge to zero as $\Delta x \to 0$.)