

Print Name:

Spring 2018 MTH 32 Test 3

Directions: Write your answers in the provided space. **Show all work.**

1. (9 pts) Evaluate the following integral: $\int \frac{x^2 + 3x - 4}{x - 2} dx$

2. (8 pts) Find the exact value of the following integral: $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x dx$

3. (9 pts) Evaluate the following integral: $\int \frac{x^3}{\sqrt{1+x^2}} dx$

4. (9 pts) Evaluate the following integral: $\int x \tan^{-1} x dx$

5. (9 pts) Evaluate the following integral: $\int \frac{x^2 + x + 4}{x^3 - 4x^2 + 4x} dx$

6. (9 pts) Determine if the integral converges or diverges. If it converges, find its value.

$$\int_1^{\infty} \frac{dx}{x^2 + x}$$

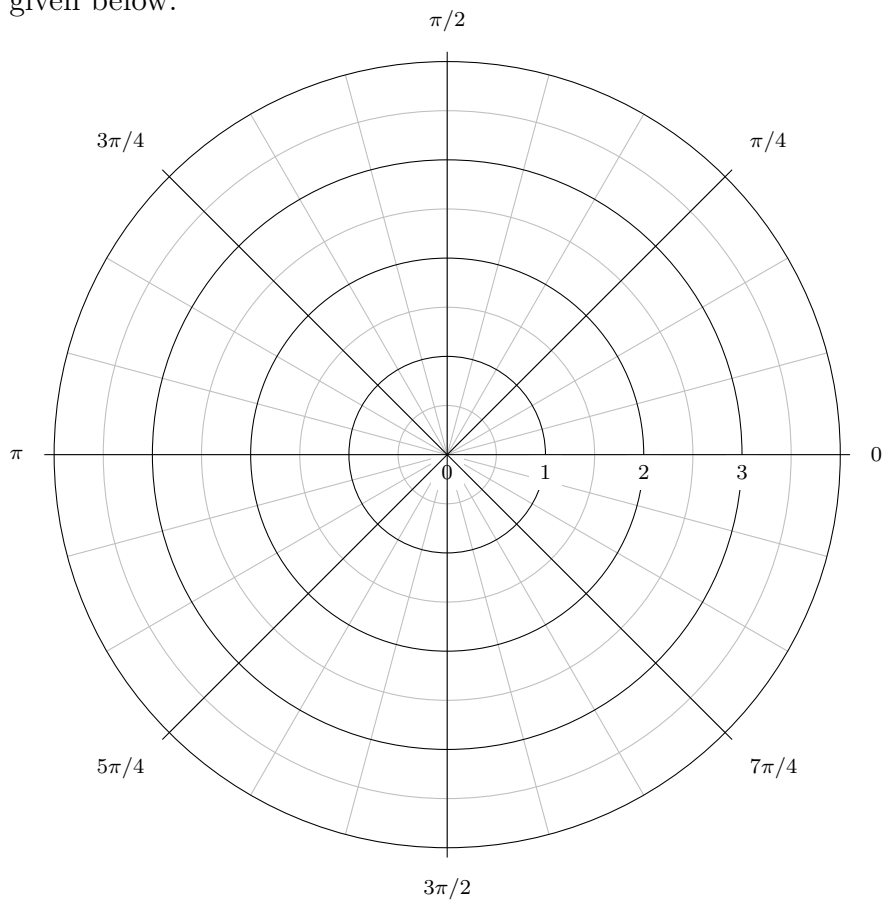
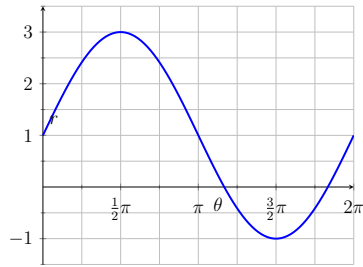
7. (9 pts) Determine if the integral converges or diverges. If it converges, find its value.

$$\int_0^1 \text{Ln } x \, dx$$

8. (9 pts) Determine if the integral converges or diverges. If it converges, find its value.

$$\int_1^{\infty} \left(\tan^{-1} x - \frac{\pi}{2} + \frac{1}{x} \right) dx$$

9. (9 pts) Sketch the polar graph of $r = 1 + 2 \sin(x\theta)$ in the axes provided. To make it easier, the cartesian graph is given below.



10. (9 pts) Find the length of the *catenary*, which is the curve with equation $y = a \cosh(x/a)$, between $x = -c$ and $x = c$. You do not need to evaluate the integral.

[The catenary is the shape that a chain takes when it hangs from two ends at the same height - *catena* means *chain* in Latin.]

11. (10 pts) Evaluate the integral: $\int \frac{5x^2 + 13x + 15}{x^3 + 4x^2 + 5x} dx$.

12. (10 pts) The *Gamma* function is defined by

$$\Gamma(p) = \int_0^{\infty} x^{p-1} e^{-x} dx.$$

It is very important in many branches of math and in statistics (it models lifespan, for example).

(a) Prove that $\Gamma(p) = (p-1) \cdot \Gamma(p-1)$.

(b) Find $\Gamma(1)$.

(c) Prove that $\Gamma(n) = (n-1)!$ when n is a positive integer. [HINT: Use (a) and (b).]

(d) Use the identity $\int_{-\infty}^{\infty} e^{-x^2} dx$ to prove that $\Gamma(1/2) = \sqrt{\pi}$.