Print Name:

Spring 2018 MTH 32 Test 2

Directions: Write your answers in the provided space. **Show all work.**

1. (12 pts) Find the derivative of the following functions. Simplify when possible.

   (a) $\tan^{-1}(\cosh x)$

   (b) $\sinh(4\sin^{-1} x + 2)$

   (c) $\sin^{-1} x + \cos^{-1} x$

   (d) $\sinh^{-1} x$

2. (12 pts) Find the exact value of

   (a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

   (b) $\sinh 2$

   (c) $\sinh^{-1}\left(\frac{e^2 - e^{-2}}{2}\right)$

   (d) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

3. (10 pts) Prove the identity: $\cosh(2x) = \cosh^2 x + \sinh^2 x$.

4. (20 pts) Evaluate the following limits, indicating the argument carefully.

(a) $\lim_{x \to 0} \dfrac{\cosh x - 1}{x}$

(b) $\lim_{x \to \infty} e^{-x} \cosh x$

(c) $\lim_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{e^x - 1} \right)$

(d) $\lim_{x \to 0} (1 - 2x)^{1/x}$

(e) $\lim_{x \to 1^+} x^{1/(x-1)}$

5. (16 pts) Evaluate the following integrals.

(a) $\displaystyle\int_0^{1/2} \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}}\, dx$ [Hint: What is $(\sin^{-1} x)'$?]

(b) $\displaystyle\int \frac{2x}{1+x^4}\, dx$ [Hint: What is $(x^2)'$?]

6. (16 pts) Evaluate the following integrals.

(a) $\displaystyle\int x \sinh x \; dx$

(b) $\displaystyle\int \sin x \sec^2 x \; dx$

7. (10 pts) Find the area between the x-axis and the graph of the function $f(x) = 9x^2 \operatorname{Ln} x$, between $x = 1$ and $x = e$.

8. (10 pts) Use integration by parts to prove the reduction formula

$$\int x^n e^x \, dx = x^n e^x - n \int x^{n-1} e^x \, dx.$$

Bonus:

1. (5 pts) What is wrong with the following argument?

   Theorem: $0 = 1$.

   Proof: Let us find $\int \frac{1}{x}\,dx$ as follows: Do integration by parts by taking

   $$u = \frac{1}{x}, \quad dv = dx \quad \text{so that} \quad du = -\frac{1}{x^2}, \quad v = x$$

   to get

   $$\int \frac{1}{x}\,dx = \frac{1}{x} \cdot x - \int -\frac{1}{x^2} \cdot x\,dx = 1 + \int \frac{1}{x}\,dx.$$

   This implies that

   $$\int \frac{1}{x}\,dx = 1 + \int \frac{1}{x}\,dx,$$

   and therefore, subtracting $\int \frac{1}{x}\,dx$ from both sides, we get

   $$0 = 1.$$

   QED

2. (5 pts) If f' is continuous, use L'Hôpital's rule to show that

   $$\lim_{h \to 0} \frac{f(x+h) - f(x-h)}{2h} = f'(x).$$

106 points total + 10 bonus