Spring 2018 MTH 32 Test 1

Directions: Write your answers in the provided space. Show all work.

1. (10 pts) Find the area of the region bounded by y = x + 4 and $y = (x - 2)^2$:



2. (10 pts) Use the washer method to find the volume of the solid obtained by rotating the region bounded by y = x + 4 and $y = (x - 2)^2$ around the x-axis. Set up the integral only. DO NOT evaluate the integral.



3. (10 pts) Use cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by y = x + 4 and $y = (x - 2)^2$ around y-axis. Set up the integral only. DO NOT evaluate the integral.



4. (10 pts) Find the area of the region bounded by $x = y^2 - 2y$ and $x = 4y - y^2$. Set up the integral only. DO NOT evaluate the integral.



5. (10 pts) Let $f(x) = x^5 + x + 3$. Show that f has an inverse function g(x) (you need to make a clear and convincing argument, using the results from class). Then find g'(21). [Hint: First show that f(x) is monotone. Then: how much is f(2)?]

6. (18 pts) Differentiate the following functions. Do not simplify.

(a)
$$f(x) = x^e + e^x$$
.
(b) $f(x) = x (\ln x)^3$

(c)
$$f(x) = \cos(e^{-x} + 1)$$
. (d) $f(x) = e^x \tan(3x + 2)$

(e)
$$f(x) = 2\sqrt{x}$$
. (f) $f(x) = \log_4 \sqrt[5]{x}$

7. (10 pts) Use logarithmic differentiation to find the derivative of the following function. You do not need to simplify.

$$f(x) = \frac{x^2\sqrt{x^2+2}}{(x+5)^4}.$$

8. (10 pts) Find the following integrals: (a) $\int x e^{x^2} dx$

(b)
$$\int \frac{1}{4x-1} dx$$

9. (12 pts) The region bounded by $y = e^x$, y = 0, x = -1 and x = 1 is revolved about the x-axis. Find the volume of the resulting solid.

Bonus:

1. (5 pts) Find the limit:

 $\lim_{x \to 0^+} x^{\frac{1}{x}}$

2. (5 pts) Given
$$x^y = y^x$$
. Find $\frac{dy}{dx}$

110 points total