## Midterm 2. Calculus I - MATH31, Section D03. Spring 2025.

Time allowed: 110 minutes Professor: Luis Fernández

NAME:\_\_\_\_\_

- The exam has SIX questions. Point values are indicated in each problem. Total is 100 points.
- Write your answers in the spaces provided. To get full credit you must show all your work.
- Please indicate your final answer clearly.
- No electronic devices besides a non-graphing calculator, or notes, are allowed.
- You will not be able to use the bathroom once the exam starts.
- 1. (32 points) Find the derivative of the following functions. Do not simplify the answer.

(a) 
$$f(x) = e^4 + x^e$$
.

(b) 
$$f(x) = \frac{\sin(2x)}{x}.$$

(c) 
$$f(x) = e^x \sqrt{x^2 + 10}$$
.

(d)  $f(x) = \ln(\sin(\cos(x^2))).$ 

(e)  $f(x) = e^{\sqrt{x}} \sin(x)$ .

(f)  $f(x) = x^{5x}$  (logarithmic differentiation).

(g)  $f(x) = \arctan(\ln x)$ 

(h)  $f(x) = \tan(x) \sinh(2x)$ 

2. (15 points) Use implicit differentiation to find an equation of the tangent line to the ellipse defined by  $x^2 - 4xy + 5y^2 = 13$  at the point (-1, -2).

**3.** (15 points) Use a linear approximation to find the approximate value of the number  $\sqrt{23}$ .

4. (20 points) he length l of a rectangle is *increasing* at a rate of 4 cm/s. At the same time its height h is *decreasing* at a rate of 7 cm/s. At what rate is the area increasing (or decreasing) when the length is 10 cm and the height is 4 cm? [The area of a rectangle of length l and height h is  $A = l \cdot h$ .]

**5.** (*8 points*) Find the following limits:

(a) 
$$\lim_{x \to 0} \frac{\sin(3x)}{\sin(2x)}$$

(b) 
$$\lim_{x \to \infty} \frac{\sinh(3x)}{e^{3x}}$$

**6.** (15 points) Suppose that f and g are differentiable functions such that  $f(g(x)) = x^3$ . We only know that g(3) = 7 and that f'(7) = -4. Find g'(3).