Midterm 1. Calculus I - MATH31, Section D03. V. 1. Spring 2025.

Time allowed: 110 minutes Professor: Luis Fernández

NAME:_____

The exam has FIVE questions. Point values are indicated in each problem, for a total of 100 points.
Write your answers in the spaces provided. To get full credit you must show all your work.
Please indicate your final answer clearly.
No calculators, electronic devices, or notes are allowed.

1. (*18 points*) Use the graph of the function f given below to find the following. Note that arrows mean that the graph extends in that direction forever, and the asymptotes are represented by dashed lines.

(a) $\lim_{x\to -2^-} f(x) =$

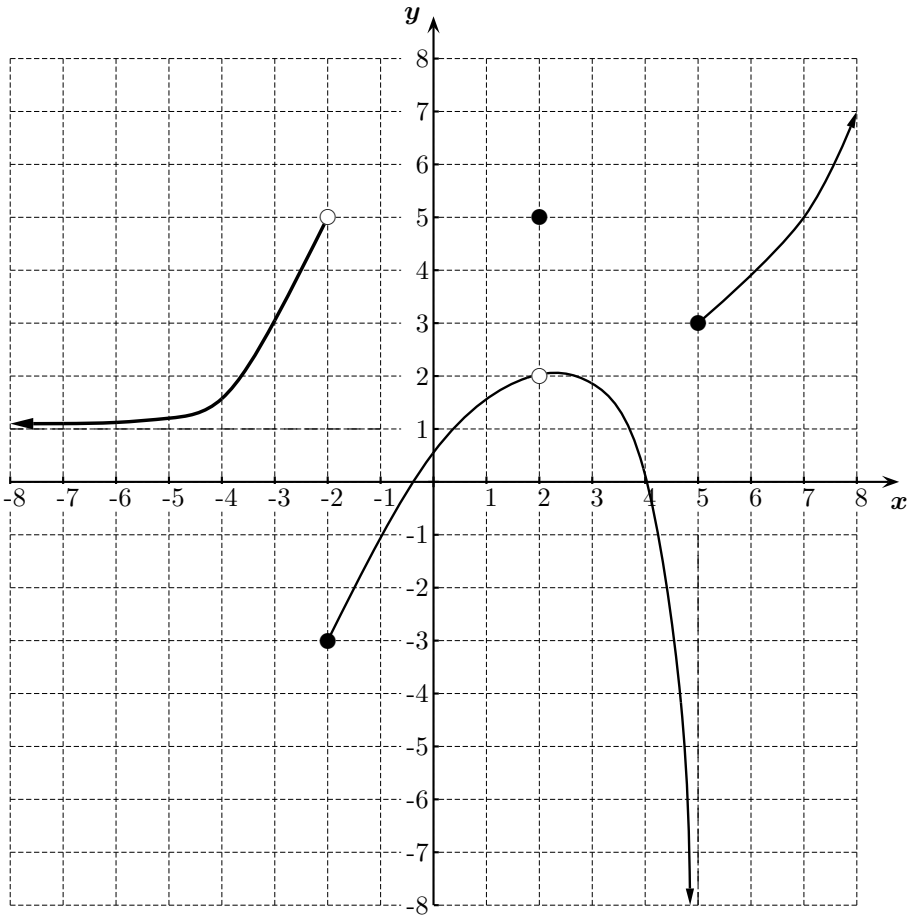
(b) $\lim_{x\to -2^+} f(x) =$

(c) $\lim_{x\to 5^-} f(x) =$

(d) $\lim_{x\to 5^+} f(x) =$

(e) $\lim_{x\to -\infty} f(x) =$

(f) At $x = -2$, is f continuous from the right, continuous from the left, or neither?
Answer:



2. (*25 points*) Find the following limits, justifing your answer.

(a) $\lim_{x\to 4} \dfrac{x^2-3x-4}{x^2-7x+12} =$

(b) $\lim_{x\to \infty} \dfrac{3x^2+2x-5}{2x^2-x+2} =$

(c) $\lim_{x\to 2^-} \dfrac{2x}{x^2-4} =$

(d) $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x^2}\right) =$

(e) $\lim_{x \to 1} \frac{\sqrt{x+8} - 3}{x^2 - 1} =$

3. (*17 points*) The figure below represents the graph of a function f, together with the tangent lines to the graph at the points with $x = -6, -4, -2, 1,$ and 4.

(a) (*2 points*) Find $f'(-6)$. Answer:

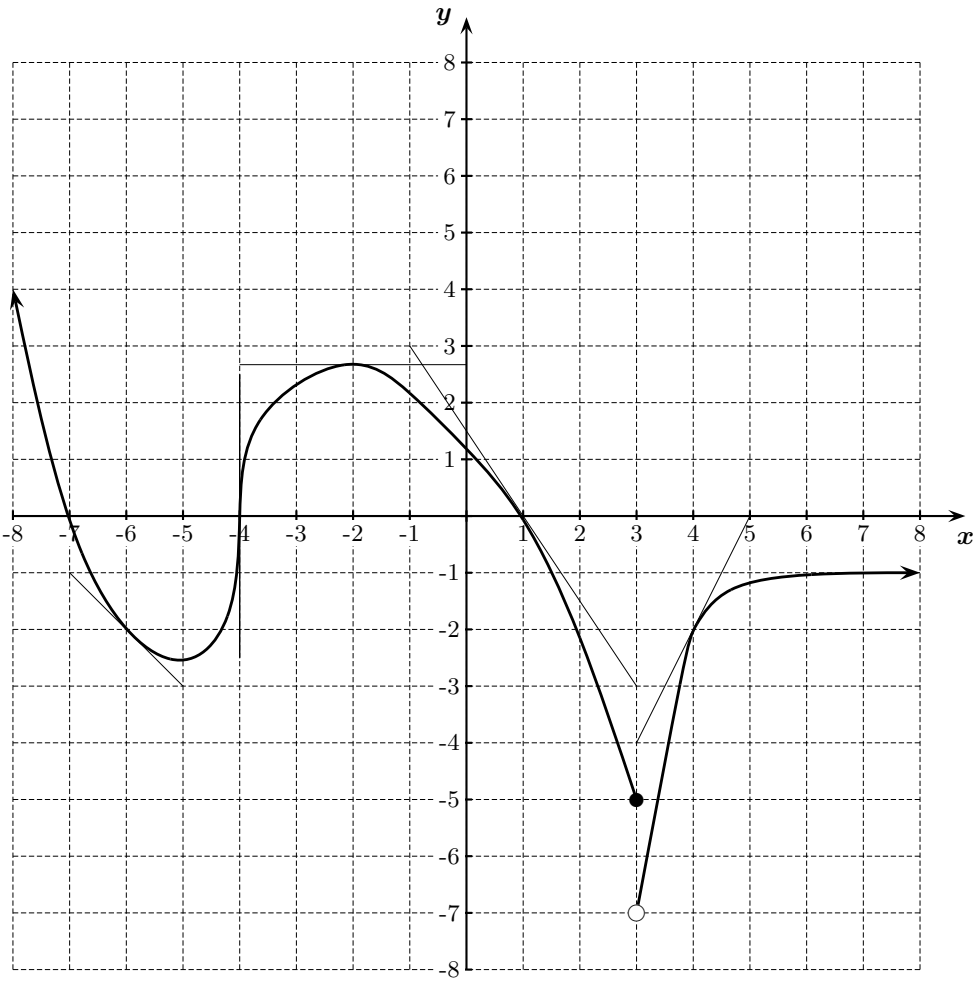
(b) (*2 points*) Find $f'(-2)$. Answer:

(c) (*2 points*) Find $f'(1)$. Answer:

(d) (*2 points*) Find $f'(4)$. Answer:

(e) (*2 points*) Find the values of x where $f'(x)$ does not exist. Answer:

(f) (*7 points*) Sketch the graph of $f'(x)$ in the same coordinate system.



4. (*20 points*) Let $f(x) = \dfrac{x+5}{x+1}$.

(a) (*5 points*) Write the definition of the derivative of a function f at the point b:

$$f'(b) = \lim$$

(b) (*10 points*) Use the definition of derivative to find $f'(1)$.

(c) (*5 points*) Use part (b) to find the equation of the tangent line to the graph of f at the point (1,3).

5. (*20 points*) Given the function $g(x) = \begin{cases} ax^2 - 4 & \text{if } x < 2 \\ x + a & \text{if } x \geq 2 \end{cases}$,

find the value of a so that g is continuous at $x = 2$.

6. BONUS EXERCISE. (*5 points*) Attempt ONLY when you have finished all the other exercises.

Let f be a continuous odd function. What is the value of $f(0)$? **Prove** carefully that the value you wrote is correct.