

**MATH 31 - Calculus. Homework 3. Due Th. 09/23/2026.** Professor Luis Fernández

**Solution: SOLUTION**

**Do not write your answers here.**

Write your answers in other sheets and **STAPLE them to this one.**

1. Let  $r(x) = f(g(h(x)))$ , where  $h(1) = 2$ ,  $g(2) = 3$ ,  $h'(1) = 2$ ,  $g'(2) = 5$  and  $f'(3) = 6$ . Find  $r'(1)$ .

**Solution:**  $r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) = f'(g(2)) \cdot g'(2) \cdot 2 = f'(3) \cdot 5 \cdot 2 = 6 \cdot 5 \cdot 2 = 60$ .

2. Evaluate, justifying your answer:

(a)  $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right)$  [HINT: do a change of variable  $t = \frac{1}{x}$ .]

(b)  $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

**Solution:**

(a) Let  $t = 1/x$ . Then, when  $x \rightarrow \infty$ ,  $t \rightarrow 0$ . Also,  $x = 1/t$ . Writing the limit using  $t$  instead of  $x$  we get  $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = \lim_{t \rightarrow 0} \frac{1}{t} \sin(t) = \lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$

(b)  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \lim_{x \rightarrow 0} \frac{\sin x}{x \cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1$ .

3. If  $g(x) = xf(x)$ , where  $f(3) = 4$  and  $f'(3) = -2$ , find an equation of the tangent line to the graph of  $g$  at the point where  $x = 3$ .

**Solution:**  $g(3) = 3 \cdot f(3) = 3 \cdot 4 = 12$ , and  $g'(x) = f(x) + xf'(x)$ , so  $g'(3) = f(3) + 3 \cdot f'(3) = 4 + 3 \cdot (-2) = 4 - 6 = -2$ .

Therefore the equation of the tangent line to the graph of  $g$  when  $x = 3$  is  $y - 12 = -2(x - 3)$ .