

MATH 31 - Calculus. Homework 2. Due Th. 09/16/2026. Professor Luis Fernández
SOLUTION

Do not write your answers here.

Write your answers in other sheets and **STAPLE them to this one.**

1. Stewart 9th edition, early transcendentals, Exercise 3 of section 2.5.
 - (a) Discontinuous at -4 , -2 , 2 and 4 . At -4 the function is not defined. At -2 it has a jump, plus the function is not defined. At 2 , it has a jump. At 4 , the limit from the left is $-\infty$.
 - (b) At -4 is not continuous from the left or from the right because the function is not defined at -4 . At -2 is continuous from the left. At 2 and 4 it is continuous from the right.
2. Stewart 9th edition, early transcendentals, Exercise 24 of section 2.6.

We have to find $\lim_{x \rightarrow \infty} \frac{t+3}{\sqrt{2t^2-1}}$. If we do the limit of the numerator and the denominator separately we get $\frac{\infty}{\infty}$, which does not tell us anything.

Therefore we have to divide numerator and denominator by the t^n , where n is the degree of the denominator. Notice that the denominator has a square root of t^2 , so its degree is actually 1. Therefore,

$$\begin{aligned}\lim_{x \rightarrow \infty} \frac{t+3}{\sqrt{2t^2-1}} &= \lim_{x \rightarrow \infty} \frac{\frac{t+3}{t}}{\frac{\sqrt{2t^2-1}}{t}} \\ &= \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{t}}{\frac{\sqrt{2t^2-1}}{\sqrt{t^2}}} \\ &= \lim_{x \rightarrow \infty} \frac{1 + \frac{3}{t}}{\sqrt{2 - \frac{1}{t^2}}} \\ &= \frac{1+0}{\sqrt{2-0}} = \frac{1}{\sqrt{2}}\end{aligned}$$

3. Find the value of a that makes f continuous everywhere.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - 4x + 3 & \text{if } 2 \leq x < 3 \end{cases}$$

The function is continuous away from the point $x = 2$ because polynomials and rational functions are continuous in their domains.

At $x = 2$, we need to find the condition for a so that $\lim_{x \rightarrow 2^-} = \lim_{x \rightarrow 2^+} = f(2)$.

$$\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{x^2-4}{x-2} = \lim_{x \rightarrow 2^-} \frac{(x-2)(x+2)}{x-2} = \lim_{x \rightarrow 2^-} (x+2) = 4.$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (ax^2 - 4x + 3) = a(2)^2 - 4(2) + 3 = 4a - 5.$$

$$f(2) = a(2)^2 - b(2) + 3 = 4a - 5.$$

The condition $\lim_{x \rightarrow 2^-} = \lim_{x \rightarrow 2^+} = f(2)$ gives $4a - 5 = 4$, which implies $4a = 9$, or $a = \frac{9}{4}$.

Thus, the function is continuous everywhere when $a = \frac{9}{4}$.