MATH 31 - Calculus. Homework 3. Due Th. 03/06/2025. Professor Luis Fernández

Solution: SOLUTION

Do not write your answers here.
Write your answers in other sheets and **STAPLE them to this one.**

1. Let $r(x) = f(g(h(x)))$, where $h(1) = 2$, $g(2) = 3$, $h'(1) = 2$, $g'(2) = 5$ and $f'(3) = 6$. Find $r'(1)$.
Solution: $r'(1) = f'(g(h(1))) \cdot g'(h(1)) \cdot h'(1) = f'(g(2)) \cdot g'(2) \cdot 2 = f'(3) \cdot 5 \cdot 2 = 6 \cdot 5 \cdot 2 = 60$.

2. Evaluate, justifying your answer:

(a) $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right)$ [HINT: do a change of variable $t = \frac{1}{x}$.]

(b) $\lim_{x\to 0} \frac{\tan x}{x}$

Solution:

(a) Let $t = 1/x$. Then, when $x \to \infty$, $t \to 0$. Also, $x = 1/t$. Writing the limit using t instead of x we get $\lim_{x\to\infty} x \sin\left(\frac{1}{x}\right) = \lim_{t\to 0} \frac{1}{t} \sin(t) = \lim_{t\to 0} \frac{\sin t}{t} = 1$

(b) $\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\sin x}{x \cos x} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot 1 = 1$.

3. If $g(x) = xf(x)$, where $f(3) = 4$ and $f'(3) = -2$, find an equation of the tangent line to the graph of g at the point where $x = 3$.

Solution: $g(3) = 3 \cdot f(3) = 3 \cdot 4 = 12$, and $g'(x) = f(x) + xf'(x)$, so $g'(3) = f(3) + 3 \cdot f'(3) = 4 + 3 \cdot (-2) = 4 - 6 = -2$.

Therefore the equation of the tangent line to the graph of g when $x = 3$ is $y - 12 = -2(x - 3)$.