MATH 31 - Calculus. Homework 2. Due Th. 02/20/2025. Professor Luis Fernández

SOLUTION

Do not write your answers here.
Write your answers in other sheets and **STAPLE them to this one.**

1. Find an equation of the tangent line to the graph of the function $f(x) = \dfrac{x+2}{x-3}$ at the point $(2,-4)$.

The slope of the tangent line at $x = 2$ is $f'(2)$. Let us calculate $f'(2)$:

$$\begin{aligned} f'(2) &= \lim_{x\to 2} \frac{\frac{x+2}{x-3} - (-4)}{x-2} \\ &= \lim_{x\to 2} \frac{\frac{x+2}{x-3} + \frac{4}{1}}{x-2} \\ &= \lim_{x\to 2} \frac{\frac{(x+2)}{x-3} + \frac{4(x-3)}{x-3}}{x-2} \\ &= \lim_{x\to 2} \frac{\frac{(x+2)+4(x-3)}{x-3}}{x-2} \\ &= \lim_{x\to 2} \frac{5x-10}{(x-3)(x-2)} \\ &= \lim_{x\to 2} \frac{5(x-2)}{(x-3)(x-2)} \\ &= \lim_{x\to 2} \frac{5}{x-3} \\ &= \frac{5}{-1} = -5 \end{aligned}$$

Therefore the equation of the tangent line is $y + 4 = -5(x-2)$.

2. Stewart 9th edition, early transcendentals, Exercise 24 of section 2.6.

We have to find $\lim_{x\to\infty} \dfrac{t+3}{\sqrt{2t^2-1}}$. If we do the limit of the numerator and the denominator separately we get $\frac{\infty}{\infty}$, which does not tell us anything.

Therefore we have to divide numerator and denominator by the t^n, where n is the degree of the denominator. Notice that the denominator has a square root of t^2, so its degree is actually 1. Therefore,

$$\begin{aligned} \lim_{x\to\infty} \frac{t+3}{\sqrt{2t^2-1}} &= \lim_{x\to\infty} \frac{\frac{t+3}{t}}{\frac{\sqrt{2t^2-1}}{t}} \\ &= \lim_{x\to\infty} \frac{1+\frac{3}{t}}{\frac{\sqrt{2t^2-1}}{\sqrt{t^2}}} \\ &= \lim_{x\to\infty} \frac{1+\frac{3}{t}}{\sqrt{2-\frac{1}{t^2}}} \\ &= \frac{1+0}{\sqrt{2-0}} = \frac{1}{\sqrt{2}} \end{aligned}$$

3. Find the value of a that makes f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - 4x + 3 & \text{if } 2 \le x < 3 \end{cases}$$

The function is continuous away from the point $x = 2$ because polynomials and rational functions are continuous in their domains.

At $x = 2$, we need to find the condition for a so that $\lim_{x\to 2^-} = \lim_{x\to 2^+} = f(2)$.

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \dfrac{x^2-4}{x-2} = \lim\limits_{x\to 2^-} \dfrac{(x-2)(x+2)}{x-2} = \lim\limits_{x\to 2^-} (x+2) = 4.$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (ax^2 - 4x + 3) = a(2)^2 - 4(2) + 3 = 4a - 5.$

$f(2) = a(2)^2 - b(2) + 3 = 4a - 5.$

The condition $\lim_{x\to 2^-} = \lim_{x\to 2^+} = f(2)$ gives $4a - 5 = 4$, which implies $4a = 9$, or $a = \frac{9}{4}$.

Thus, the function is continuous everywhere when $a = \frac{9}{4}$.