

Basic Properties of Definite Integrals

Let f and g be integrable functions on the interval $[a, b]$, and let c be a constant. Then:

1. **Most basic integral (interpret as area):**

$$\int_a^b 1 \cdot dx = (b - a) \quad (\text{the integral of the constant function } 1).$$

2. **Linearity:**

$$\int_a^b (cf(x)) dx = c \int_a^b f(x) dx$$

$$\int_a^b (f(x) + g(x)) dx = \int_a^b f(x) dx + \int_a^b g(x) dx$$

$$\int_a^b (f(x) - g(x)) dx = \int_a^b f(x) dx - \int_a^b g(x) dx$$

3. **Reversal of limits:**

$$\int_a^b f(x) dx = - \int_b^a f(x) dx$$

4. **Zero-width interval:**

$$\int_a^a f(x) dx = 0$$

5. **Additivity over intervals:**

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$

6. **Comparison properties:**

$$\text{If } f(x) \geq 0 \text{ for all } x \in [a, b], \text{ then } \int_a^b f(x) dx \geq 0$$

$$\text{If } f(x) \geq g(x) \text{ for all } x \in [a, b], \text{ then } \int_a^b f(x) dx \geq \int_a^b g(x) dx$$

$$\text{If } m \leq f(x) \leq M \text{ for all } x \in [a, b], \text{ then } m(b - a) \leq \int_a^b f(x) dx \leq M(b - a)$$

Fundamental Theorem of Calculus

Part 1

If f is continuous in $[a, b]$, then $\frac{d}{dx} \int_a^x f(t) dt = f(x)$ for x in $[a, b]$.

In particular, the function $g(x) = \int_a^x f(t) dt$ is differentiable for x in $[a, b]$.

Part 2

If f is continuous in $[a, b]$, and if F is an antiderivative of f , then

$$\int_a^b f(x) dx = F(b) - F(a).$$

Or, stated differently,

$$\int_a^b F'(x) dx = F(b) - F(a).$$

(The book calls this last statement “The net change theorem”.)