

## Chain rule exercises to do in class

1. Differentiate the following functions using the chain rule:

(a)  $f(x) = \sin(2x^3 + 5x)$

(b)  $f(x) = e^{4x^2+3x}$

(c)  $f(x) = \cos(3x^2 - 2x)$

(d)  $f(x) = (5x^2 - x)^4$

(e)  $f(x) = \sqrt{6x^4 + 2x^2}$

(f)  $f(x) = \tan(3x^3 + 2x)$

(g)  $f(x) = \sin(5x^4 - 3x^3)$

(h)  $f(x) = (4x^3 - 2x)^5$

(i)  $f(x) = e^{2x^2+3x}$

(j)  $f(x) = \cos(6x^2 + 3x)$

2. Compute the derivative of the following functions:

(a)  $f(x) = \tan(4x^2 + x)$

(b)  $f(x) = \sin(3x^4 - 2x^3)$

(c)  $f(x) = (2x^3 + 3x^2 - x)^5$

(d)  $f(x) = e^{x^2+1}$

(e)  $f(x) = (x^3 + 2x)^7$

(f)  $f(x) = \sqrt{3x^5 - x^3 + 2x}$

(g)  $f(x) = \cos(2x^4 - 5x^2)$

(h)  $f(x) = e^{5x^4-3x^2}$

(i)  $f(x) = (3x^2 - 4x)^3$

(j)  $f(x) = \sin(2x^2 + 3x)$

3. Differentiate the following compositions:

(a)  $f(x) = \sqrt{7x^3 - 2x}$

(b)  $f(x) = \cos(5x^2 + 4x)$

(c)  $f(x) = (x^4 - x^2)^6$

(d)  $f(x) = e^{3x^3-2x}$

(e)  $f(x) = \sin(4x^2 - x)$

(f)  $f(x) = (6x^3 - x^2 + 5)^4$

(g)  $f(x) = \sqrt{5x^4 + 3x^2}$

(h)  $f(x) = \cos(4x^3 + 3x^2)$

(i)  $f(x) = e^{7x^2+4x}$

(j)  $f(x) = (x^2 - 1)^5$

4. Find the second derivative of the following functions:

(a)  $f(x) = (3x^2 - x)^5$

(b)  $f(x) = \sqrt{2x^4 + 5x^2}$

## Solutions:

1. Differentiate the following functions using the chain rule:

$$(a) \quad f(x) = \sin(2x^3 + 5x) \qquad f'(x) = \cos(2x^3 + 5x) \cdot (6x^2 + 5)$$

$$(b) \quad f(x) = e^{4x^2+3x} \qquad f'(x) = e^{4x^2+3x} \cdot (8x + 3)$$

$$(c) \quad f(x) = \cos(3x^2 - 2x) \qquad f'(x) = -\sin(3x^2 - 2x) \cdot (6x - 2)$$

$$(d) \quad f(x) = (5x^2 - x)^4 \qquad f'(x) = 4(5x^2 - x)^3 \cdot (10x - 1)$$

$$(e) \quad f(x) = \sqrt{6x^4 + 2x^2} \qquad f'(x) = \frac{1}{2\sqrt{6x^4 + 2x^2}} \cdot (24x^3 + 4x)$$

$$(f) \quad f(x) = \tan(3x^3 + 2x) \qquad f'(x) = \sec^2(3x^3 + 2x) \cdot (9x^2 + 2)$$

$$(g) \quad f(x) = \sin(5x^4 - 3x^3) \qquad f'(x) = \cos(5x^4 - 3x^3) \cdot (20x^3 - 9x^2)$$

$$(h) \quad f(x) = (4x^3 - 2x)^5 \qquad f'(x) = 5(4x^3 - 2x)^4 \cdot (12x^2 - 2)$$

$$(i) \quad f(x) = e^{2x^2+3x} \qquad f'(x) = e^{2x^2+3x} \cdot (4x + 3)$$

$$(j) \quad f(x) = \cos(6x^2 + 3x) \qquad f'(x) = -\sin(6x^2 + 3x) \cdot (12x + 3)$$

2. Compute the derivative of the following functions:

$$(a) \quad f(x) = \tan(4x^2 + x) \qquad f'(x) = \sec^2(4x^2 + x) \cdot (8x + 1)$$

$$(b) \quad f(x) = \sin(3x^4 - 2x^3) \qquad f'(x) = \cos(3x^4 - 2x^3) \cdot (12x^3 - 6x^2)$$

$$(c) \quad f(x) = (2x^3 + 3x^2 - x)^5 \qquad f'(x) = 5(2x^3 + 3x^2 - x)^4 \cdot (6x^2 + 6x - 1)$$

$$(d) \quad f(x) = e^{x^2+1} \qquad f'(x) = e^{x^2+1} \cdot 2x$$

$$(e) \quad f(x) = (x^3 + 2x)^7 \qquad f'(x) = 7(x^3 + 2x)^6 \cdot (3x^2 + 2)$$

$$(f) \quad f(x) = \sqrt{3x^5 - x^3 + 2x} \qquad f'(x) = \frac{1}{2\sqrt{3x^5 - x^3 + 2x}} \cdot (15x^4 - 3x^2 + 2)$$

$$(g) \quad f(x) = \cos(2x^4 - 5x^2) \qquad f'(x) = -\sin(2x^4 - 5x^2) \cdot (8x^3 - 10x)$$

$$(h) f(x) = e^{5x^4-3x^2}$$

$$f'(x) = e^{5x^4-3x^2} \cdot (20x^3 - 6x)$$

$$(i) f(x) = (3x^2 - 4x)^3$$

$$f'(x) = 9(3x^2 - 4x)^2 \cdot (6x - 4)$$

$$(j) f(x) = \sin(2x^2 + 3x)$$

$$f'(x) = \cos(2x^2 + 3x) \cdot (4x + 3)$$

3. Differentiate the following compositions:

$$(a) f(x) = \sqrt{7x^3 - 2x}$$

$$f'(x) = \frac{1}{2\sqrt{7x^3 - 2x}} \cdot (21x^2 - 2)$$

$$(b) f(x) = \cos(5x^2 + 4x)$$

$$f'(x) = -\sin(5x^2 + 4x) \cdot (10x + 4)$$

$$(c) f(x) = (x^4 - x^2)^6$$

$$f'(x) = 6(x^4 - x^2)^5 \cdot (4x^3 - 2x)$$

$$(d) f(x) = e^{3x^3-2x}$$

$$f'(x) = e^{3x^3-2x} \cdot (9x^2 - 2)$$

$$(e) f(x) = \sin(4x^2 - x)$$

$$f'(x) = \cos(4x^2 - x) \cdot (8x - 1)$$

$$(f) f(x) = (6x^3 - x^2 + 5)^4$$

$$f'(x) = 4(6x^3 - x^2 + 5)^3 \cdot (18x^2 - 2x)$$

$$(g) f(x) = \sqrt{5x^4 + 3x^2}$$

$$f'(x) = \frac{1}{2\sqrt{5x^4 + 3x^2}} \cdot (20x^3 + 6x)$$

$$(h) f(x) = \cos(4x^3 + 3x^2)$$

$$f'(x) = -\sin(4x^3 + 3x^2) \cdot (12x^2 + 6x)$$

$$(i) f(x) = e^{7x^2+4x}$$

$$f'(x) = e^{7x^2+4x} \cdot (14x + 4)$$

$$(j) f(x) = (x^2 - 1)^5$$

$$f'(x) = 5(x^2 - 1)^4 \cdot 2x$$

4. Find the second derivative of the following functions:

$$(a) f(x) = (3x^2 - x)^5$$

$$f'(x) = 5(3x^2 - x)^4 \cdot (6x - 1)$$

$$f''(x) = 5 [4(3x^2 - x)^3 \cdot (6x - 1)^2 + (3x^2 - x)^4 \cdot 12x]$$

$$(b) f(x) = \sqrt{2x^4 + 5x^2}$$

$$f'(x) = \frac{1}{2\sqrt{2x^4 + 5x^2}} \cdot (8x^3 + 10x)$$

$$f''(x) = \frac{1}{2\sqrt{2x^4 + 5x^2}} \cdot \left[ (24x^2 + 10) \cdot \sqrt{2x^4 + 5x^2} - (8x^3 + 10x) \cdot \frac{8x^3 + 10x}{\sqrt{2x^4 + 5x^2}} \right]$$