

Midterm 2. Calculus I - MATH31, Section D03. Practice test. Spring 2025.

Time allowed: 110 minutes Professor: Luis Fernández

NAME: _____

- The exam has FIVE questions. Point values are indicated in each problem, for a total of 100 points.
- Write your answers in the spaces provided. To get full credit you must show all your work.
- Please indicate your final answer clearly.
- No electronic devices besides a non-graphing calculator, or notes, are allowed.
- You will not be able to use the bathroom once the exam starts.

1. (32 points) Find the derivative of the following functions. You do not need to simplify the answer.

(a) $f(x) = \pi^4 + x^\pi$.

(e) $e^{\sqrt{x}} \cos(x)$.

(b) $f(x) = \frac{\sin(2x)}{1 + \cos(2x)}$.

(f) $f(x) = x^x$ (logarithmic differentiation).

(c) $f(x) = (x^2 + 10)e^{2x}$.

(g) $f(x) = \frac{1}{\arctan(2x + 5)}$

(d) $f(x) = \ln(x + \sqrt{x^2 - 1})$.

(h) $f(x) = e^{2x} \sinh 2x$

2. (15 points) Use implicit differentiation to find an equation of the tangent line to the ellipse defined by $3x^2 + 4xy + 5y^2 = 37$ at the point $(4, -1)$.

3. (15 points) Find the linearization $L(x)$ of the function $g(x) = xf(x^2)$ at $x = 2$ given the following information $f(2) = -1, f'(2) = 4, f(4) = 5, f'(4) = -2$.

4. (20 points) The radius r of a cylinder with base and lid is *increasing* at a rate of 2 cm/s. At the same time its height h is *decreasing* at a rate of 5 cm/s. At what rate is the area increasing (or decreasing) when the radius is 5 cm and the height is 10 cm? [The area of a cylinder with base and lid, of radius r and height h is given by $A = 2\pi rh + 2\pi r^2$.]

5. (8 points) Find the following limits:

(a) $\lim_{x \rightarrow 0} \frac{\sin(4x)}{\sin(5x)}$

(b) $\lim_{x \rightarrow \infty} \frac{\sinh(4x)}{\cosh(5x)}$

6. (15 points) Suppose that f and g are differentiable functions such that $f(g(x)) = x$. We only know that $g(3) = 5$ and that $f'(5) = -12$. Find $g'(3)$.