Spring 2025 MTH 31 Test 1 Review

Our Test 1 will be given in class on Feb 19. Please do not wait until the last minute to prepare for the test.

Sections: 2.2/2.3/2.5/2.6/2.7/2.8:

Section 2.2:

- Understand the definition of a limit and one-sided limits
- Understand that the limit of $f(x)$ as $x \to a$ equals L if and only if the limits from the left and right both equal L.
- Know how to find infinite limits and vertical asymptotes

Section 2.3:

- Know the basis limit laws (know when to apply them): Sum, difference, constant multiple, product, quotient, power, root laws etc
- Be able to apply the laws to calculate different types of limits
- Know the direct substitution property

Section 2.5:

- Know the definition of continuity (implicitly has 3 conditions)
- Know common continuous functions
- Be able to determine the type of discontinuities (focus on removable/jump/infinite discontinuities)
- Be able to state the Intermediate Value Theorem and use it to show existence of solution of a polynomial equation on an interval.
- Know the definition of horizontal asymptote and how to find horizontal asymptotes

Section 2.6:

- Know the definition of limits at infinity and how to find limits at infinity
- Know the definition of horizontal asymptote and how to find horizontal asymptotes

Section 2.7:

- Know the limit definition of derivative of a function at a point
- Understand the physical meaning (instantaneous change) and geometric meaning (slope of the tangent line) of a derivative
- Be able to find the equation of the tangent line of a function at a point using the limit definition

Section 2.8:

- Know the limit definition of derivative of a function as a function
- Be able to derive the derivative of a function using the limit definition (including the application in physics – e.g. the velocity function as a derivative of the position function)
- Understand when a function fails to be differentiable (e.g. f is not continuous at a (hole, asymptote, or jump etc)); f has a cusp/corner at a;f has a vertical tangent at a.
- Know that if f is differentiable at a then it is necessarily continuous at a, but not vice versa
- Be able to tell when the function is not differentiable based on the graph of a function

Review Problems:

1. Answer the following True-False questions.

___ a. The limit of a function always exists at every point in its domain.

___ b. It is possible that $\lim_{x\to 2} f(x) = 5$ but $f(2) = 4$.

___ c. If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a}[f(x) + g(x)]$ does not exist.

___ d. $\lim_{x\to a}[f(x)g(x)]$ may exist though neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists.

___ e. If a function is continuous at a point, then it must be differentiable at that point..

___ f. Differentiable functions are always continuous.

___ g. The limit $\lim_{h\to 0} \dfrac{(x+2)^2 - 4}{h}$ represents the derivative of $f(x) = x^2$ at $x = 4$.

___ h. If $f(x)$ is continuous, then $|f(x)|$ is continuous.

___ i. If $|f(x)|$ is continuous, then $f(x)$ is continuous.

___ j. If a function f is continuous everywhere, and if $f(0) = -2$ and $f(4) = 1$, then $f(x)$ must have a root somewhere in $(0, 4)$.

___ k. If the graph of f has a vertical asymptote at $x = 5$, then $\lim_{x\to 5} f(x) = \infty$.

___ l. If $\lim_{x\to 2} f(x) = -\infty$, then the graph of f has a vertical asymptote at $x = 2$.

___ m. If a rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 2$, then the degree of $p(x)$ is equal to the degree of $q(x)$.

___ n. If the degree of the polynomial $p(x)$ is less than the degree of the polynomial $q(x)$, then there are no horizontal asymptotes for the rational function $\frac{p(x)}{q(x)}$

___ o. If $f(2) = 5$, $f'(2) = 4$ for a differentiable function $f(x)$, then the line tangent to the graph of $f(x)$ at $x = 2$ has slope 4 and passes the point $(2, 5)$.

2. Evaluate the following limits. If a limit doesn't exist, state so and explain why.

(a) $\displaystyle\lim_{x\to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$ (b) $\displaystyle\lim_{x\to\infty} \frac{x^2 - 4}{x^2 - 5x + 6}$ (c) $\displaystyle\lim_{x\to 2} \frac{3x - 6}{|x - 2|}$

(d) $\displaystyle\lim_{x\to 2^-} \frac{-5}{(x-2)^3}$ (e) $\displaystyle\lim_{x\to\infty} (\sqrt{9x^2 - 5x} - 3x)$ (f) $\displaystyle\lim_{t\to 0} \left(\frac{7}{t\sqrt{49 + t}} - \frac{1}{t}\right)$

(g) $\displaystyle\lim_{h\to 0} \frac{\frac{2}{(a+h)^2} - \frac{2}{a^2}}{h}$ (h) $\displaystyle\lim_{x\to 0} \cos(\pi e^x)$ (i) $\displaystyle\lim_{x\to \frac{\pi}{4}} \ln(\tan x)$

(j) $\displaystyle\lim_{x\to 0} x^2 \cos\frac{5}{x}$ (k) $\displaystyle\lim_{x\to\infty} e^{-x}\sin x$ (l) $\displaystyle\lim_{x\to 0^+} \sqrt{x}e^{\cos\frac{\pi}{x}}$

3. Determine the point(s) at which each of the following function is discontinuous and state the type of discontinuity: removable, jump, infinite, or none of these.

(a) $\dfrac{x-1}{x^2-4x+3}$ (b) $\dfrac{x-2}{|x-1|}$ (c) $\dfrac{4x-4}{|x-1|}$

4. Let

$$f(x) = \begin{cases} ax^2 - 3x + 4 & \text{if } x \le 2 \\ x + 3a & \text{if } x > 2 \end{cases}$$

Find the real number a so that f is continuous on $(-\infty, \infty)$.

5. State the Intermediate Value Theorem. Show that each of the following equations has a solution in the given interval.
   (a) $x^4 - 2x^3 + 3x^2 - 2x - 6 = 0$ in $(-1, 1)$
   (b) $2^x = x^2$ in $(-1, 0)$
   (c) $\cos x = x$ in $\left(0, \dfrac{\pi}{2}\right)$

6. Calculate the derivative of each of the following functions using the definition of the derivative as the limit of the difference quotients. Then find an equation of the tangent line to the curve at the given point:
   (a) $f(x) = x^2 + 5x - 2$, at the point $x = 2$
   (b) $f(x)y = \dfrac{x+1}{x+2}$, at the point $x = 1$.
   (c) $f(x) = \sqrt{2x-1}$, at the point $(5, 3)$.

7. The equation of motion of a particle is $s(t) = t^2 - 5t$, where s is in meters and t is in seconds. Assuming that $t \ge 0$, answer the following questions.

   (a) Find the velocity function $v(t)$ (b) Find the velocity after 3 seconds.

8. Use the given graph of the function f to find the x-values for which f is not differentiable. Explain why.