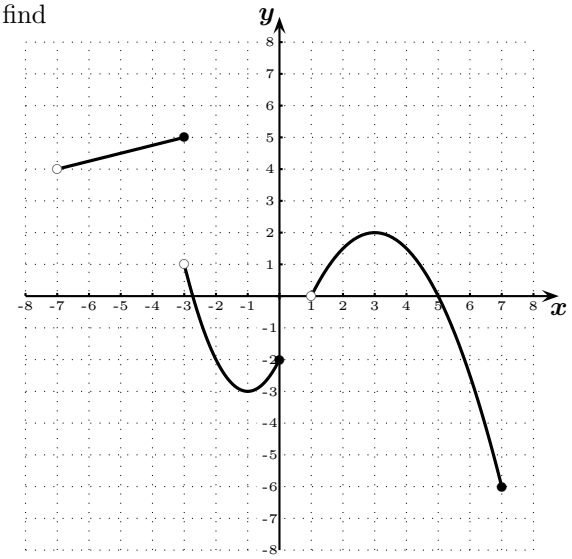
Practice Midterm 1. Calculus I - MTH31. Section D03.

Time allowed: 110 minutes Professor: Luis Fernández

NAME:_____

The exam has SIX questions. Point values are indicated in each problem, for a total of 100 points. Write your answers in the spaces provided. To get full credit you must show all your work. Please indicate your final answer clearly.

1. (*12 points*) Use the graph of the function f given below to find



(a) $\lim_{x\to -3^-} f(x) =$

(b) $\lim_{x\to -3^+} f(x) =$

(c) $\lim_{x\to 0^-} f(x) =$

(d) $\lim_{x\to 1^+} f(x) =$

(e) The domain of f.

(f) At $x = -3$, is f continuous from the right, continuous from the left, or neither?

2. (*12 points*) Show that the equation $x^5 - 6x^4 - 5x^2 + 4 = 0$ has a solution c between 0 and 1. Explain clearly, and mention any theorem that you use.

3. (*24 points*) Find the following limits, justifing your answer.

(a) $\lim_{x\to 3} \dfrac{x^2-9}{x^2-x-6} =$

(b) $\lim_{x\to 0} \dfrac{x^2+1}{x^2} =$

(c) $\lim_{t\to 3} \dfrac{\sqrt{t+1}-2}{t-3} =$

(d) $\lim_{x\to 0} \dfrac{x^2-3x-4}{\cos x} =$

(e) $\lim_{x\to \infty} x - \sqrt{x^2+4x} =$

(f) $\lim_{x\to -\infty} \dfrac{x^2+1}{x^2+4} =$

(g) $\lim_{x\to 0} x\cos\left(\dfrac{1}{x}\right) =$

(h) $\lim_{x\to \pi} \sin\left(\dfrac{x^2-\pi^2}{x-\pi}\right) =$

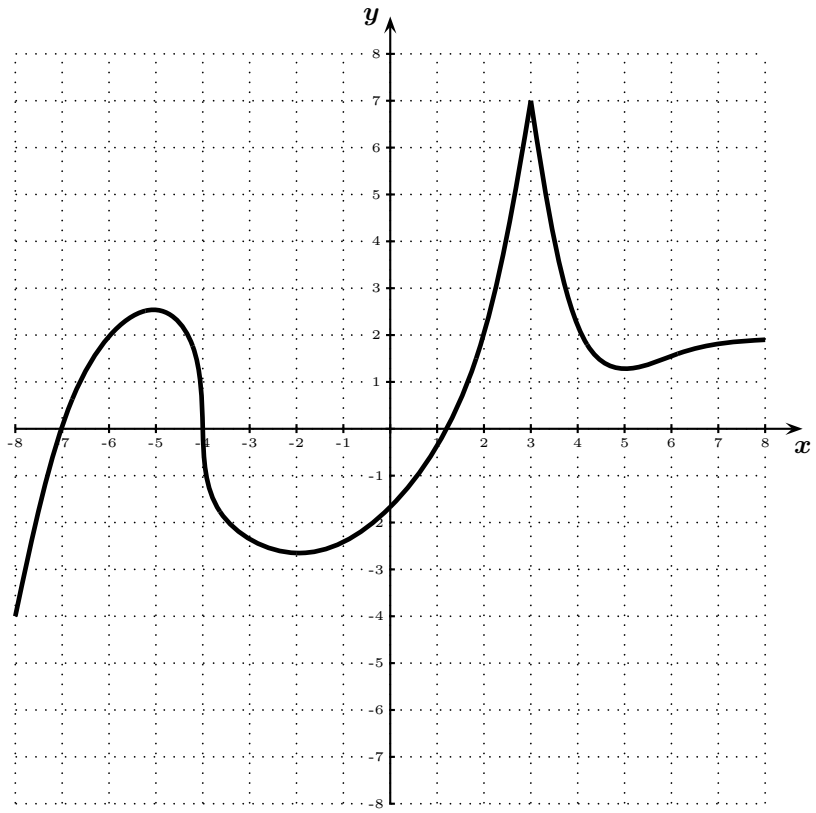
4. (*20 points*) The figure below represents the graph of a function f.

(a) (*4 points*) Find the values of x where $f'(x) = 0$. Answer:

(b) (*4 points*) Find the values of x where $f'(x)$ does not exist. Answer:

(c) (*4 points*) Find the approximate values of x where $f'(x) = 1$. Answer:

(d) (*8 points*) Sketch the graph of $f'(x)$ in the same coordinate system.



5. (a) (*8 points*) Write the definition of the derivative of a function f at a point b:

$$f'(b) = \lim$$

(b) (*8 points*) Using the definition, find $f'(0)$ for the function given by $f(x) = x\,|x|$.

6. (*16 points*) Given the function $g(x) = \begin{cases} x^2 - 1 & \text{if } x < 1 \\ 4 & \text{if } x = 1 \\ (3-x)^2 & \text{if } x > 1 \end{cases}$

(a) Determine whether the function g is continuous from the right, continous from the left, or neither, at the point $x = 1$.

7. BONUS EXERCISE. (*10 points*) Attempt ONLY when you have finished all the other exercises.

By definition, a function f is *increasing* at a point a if $f(x) - f(a)$ has the same sign as $x - a$ for x near a. Use the definition of derivative to prove that if a function f is increasing at a then $f'(a) \geq 0$.