MATH 30 - Precalculus, Version B

Second Midterm. Time allowed: 2 hours, 15 minutes. Professor Luis Fernández

NAME: SOLUTION

Instructions:

- Write all your answers in the space provided, or attach sheets if you need more space.
- **SHOW ALL YOUR WORK**. Solutions without work shown will receive no credit.
- Non-graphing calculators are allowed. No notes or books allowed.
- The exam has 8 exercises. The points of each exercise are written on the left.
- The exam has a total of 110 points, with 10 extra credit points.

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[18] **1.** Write the **exact value (NO decimals)** of

a) $\log_1 64 =$ 3

$4^3 = 64$

b) $\log_6 \sqrt[3]{6} =$ $\frac{1}{3}$

$6^{1/3}$

c) $\log_{123} 123^7 =$ 7

d) $2013^{\log_{2013} 6} =$ 6

e) $\log_9 \frac{1}{27} =$ $-\frac{3}{2}$

$9^x = \frac{1}{27} \rightarrow 3^{2x} = 3^{-3}$

$\Rightarrow 2x = -3$

$x = -\frac{3}{2}$

f) $\log_{16} 8 =$ $\frac{3}{4}$

$16^x = 8$

$\Rightarrow 2^{4x} = 2^3$

$4x = 3 \rightarrow x = \frac{3}{4}$

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[6] **2.** Convert the following from exponential form to logarithmic form.

a) $e^x = 2$

$\text{Ln}\, 2 = x$

b) $10^{x-2} = 7$

$\log 7 = x - 2$

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[6] **3.** Convert the following from logarithmic form to exponential form.

a) $\text{Ln}\, y = 3$

$e^3 = y$

b) $\log_3(y+6) = x + 4$

$3^{x+4} = y + 6$

[8] **4.** Condense the following logarithmic expressions (that is, write them using only one logarithm in the front).

a) $5\log x + 2\log y$

$\log(x^5 y^2)$

b) $7\log a + 2\log b - 7\log c$

$\log\left(\frac{a^7 b^2}{c^7}\right)$

c) $\frac{1}{5}(2\log x - \frac{1}{2}\log y + \frac{2}{3}\log z)$

$\log\sqrt[5]{\frac{x^2 z^{2/3}}{\sqrt{y}}} = \log\left(\frac{x^2 z^{2/3}}{y^{1/2}}\right)^{1/5}$ ✓ ✓

d) $\frac{\log x}{5} - \frac{4}{7}\log y$

$\log\left(\frac{x^{1/5}}{y^{4/7}}\right)$

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[8] **5.** Expand the following logarithmic expressions (that is, write them using addition and subtraction of logarithms).

a) $\log_5(4yz)$

$\log_5 4 + \log_5 y + \log_5 z$

b) $\log_7(x^2 y^1)$

$2\log_7 x + 4\log_7 y$

c) $\log_9\left(\frac{x^{14}}{4}\right)$

$14\log_9 x - \log_9 4$

d) $\log(x^1 y^3)^5$

$5(4\log x + 3\log y)$

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[4] **6.** Write the following logarithms in the indicated base.

a) $\log_5 7$, in base 6.

$\log_5 7 = \frac{\log_6 7}{\log_6 5}$

b) $\log_2 9$, in base 10.

$\log_2 9 = \frac{\log 9}{\log 2}$

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[40] **7.** Solve the following equations. If the answer is not an exact numbers, leave it expressed as a logarithm.

a) $7^{x-2} = 49$

$7^{x-2} = 7^2 \Rightarrow x-2 = 2 \Rightarrow \boxed{x=4}$

Check

$LHS = 7^{4-2} = 7^2 = 49 = RHS$ ✓

b) $4^{x-3} = 8^{2x+1}$

$2^{2(x-3)} = 2^{3(2x+1)} \Rightarrow 2(x-3) = 3(2x+1)$

$\Rightarrow 2x-6 = 6x+3 \Rightarrow -4x = 9 \Rightarrow \boxed{x = -\frac{9}{4}}$

Check:

$LHS = 4^{-\frac{9}{4}-3} = 4^{-\frac{21}{4}} = 2^{-\frac{21}{2}}$ ✓

$RHS = 8^{2(-\frac{9}{4})+1} = 8^{-\frac{9}{2}+1} = 8^{-\frac{7}{2}} = 2^{-\frac{21}{2}}$ ✓

c) $\log_2(x) - 4 = \log_2 3$

$\log_2(x) = 4 + \log_2 3 \Rightarrow \log_2(x) - \log_2(3) = 4$

$\Rightarrow \log_2\left(\frac{x}{3}\right) = 4 \Rightarrow \frac{x}{3} = 2^4 = 16 \Rightarrow \boxed{x = 48}$

Check:

$LHS = \log_2 48 - 4 = \log_2 16 + \log_2 3 - 4 = \log_2 3 = RHS$ ✓

($48 - 4 = 16 \cdot 3$; $\log_2 16 = 4$)

d) $\log_3(2x-7) + \log_3(x-2) = \log_3(2x-7)$

$\log_3((x-4)(x-2)) = \log_3(2x-7) \Rightarrow (x-4)(x-2) = 2x-7$

$\Rightarrow x^2-4x-2x+8 = 2x-7 \Rightarrow x^2-6x+8 = 2x-7 \Rightarrow x^2-8x+15 = 0$

$\Rightarrow (x-3)(x-5) = 0 \Rightarrow x = 3$ or $x = 5$.

Check: $x = 3 \rightarrow \log_3(3-4)$ NOT GOOD. $x=3$ is not a solution

$\boxed{x=5}$ $\log_3(5-4) + \log_3(5-2) \stackrel{?}{=} \log_3(5\cdot 2-7)$ ✓

($\log_3(1) = 0$; $3 = 3$)

[20] **8.** For the rational function $f(x) = \dfrac{x^2 + 2x - 3}{x^2 - 2x - 3}$,

a) Factor numerator and denominator and simplify if possible.

$$f(x) = \frac{(x+3)(x-1)}{(x-3)(x+1)}$$

b) Find the x intercepts of the graph of $y = f(x)$, if they exist.

$(x+3)(x-1) = 0 \Rightarrow$ **$x = -3$ or $x = 1$**

c) Find the y intercepts of the graph of $y = f(x)$, if they exist.

$f(0) = \frac{-3}{-3} = 1$

d) Find any vertical asymptotes.

$(x-3)(x+1) = 0 \Rightarrow$ **$x = 3$ or $x = -1$**

V.A. at $x = 3$
V.A. at $x = -1$

e) Find any horizontal asymptotes.

As $x \to \pm\infty$, $f(x) \approx \frac{x^2}{x^2} = 1$.

$\Rightarrow$ **H.A at $y = 1$**

f) Use the information above to sketch a graph of $y = f(x)$.

$$f(5) = \frac{(5+3)(5-1)}{(5-3)(5+1)} = \frac{8 \cdot 4}{2 \cdot 6} = \frac{8}{3} \approx 2.6$$