

MATH 30 - Precalculus, Version A

Second Midterm. Time allowed: 2 hours, 15 minutes. Professor Luis Fernández

NAME: SOLUTION

Instructions:

- Write all your answers in the space provided, or attach sheets if you need more space.
- **SHOW ALL YOUR WORK.** Solutions without work shown will receive no credit.
- Non-graphing calculators are allowed. No notes or books allowed.
- The exam has 8 exercises. The points of each exercise are written on the left.
- The exam has a total of 110 points, with 10 extra credit points.

[18] 1. Write the exact value (NO decimals) of

a) $\log_3 81 = \boxed{4}$

$3^4 = 81$

b) $\log_5 \sqrt[4]{5} = \boxed{\frac{1}{4}}$

$5^{1/4}$

c) $1023^{\log_{1023} 5} = \boxed{5}$

d) $\log_{4513} 4513^{13} = \boxed{13}$

e) $\log_{16} 8 = \boxed{\frac{3}{4}}$

$16^x = 8$

$2^{4x} = 2^3 \Rightarrow 4x = 3, x = \frac{3}{4}$

f) $\log_9 \frac{1}{27} = \boxed{-\frac{3}{2}}$

$9^x = \frac{1}{27} \rightarrow 3^{2x} = 3^{-3}$

$\Rightarrow 2x = -3 \Rightarrow \boxed{x = -\frac{3}{2}}$

[6] 2. Convert the following from exponential form to logarithmic form.

a) $e^x = 8$

$\boxed{\ln 8 = x}$

b) $10^{x+3} = 16$

$\boxed{\log_{10} 16 = x+3}$

[6] 3. Convert the following from logarithmic form to exponential form.

a) $\ln y = 8$

$\boxed{e^8 = y}$

b) $\log_6(y+8) = x+3$

$\boxed{6^{x+3} = y+8}$

[8] 4. Condense the following logarithmic expressions (that is, write them using only one logarithm in the front).

a) $4 \log x + 3 \log y$

$$\log(x^4 y^3)$$

b) $12 \log a - 2 \log b + 5 \log c$

$$\log\left(\frac{a^{12} c^5}{b^2}\right)$$

c) $\frac{1}{5}(2 \log x - \frac{1}{2} \log y + \frac{2}{3} \log z)$

$$\log \sqrt[5]{\frac{x^2 z^{2/3}}{\sqrt{y}}} = \log\left(\frac{x^2 z^{2/3}}{y^{1/2}}\right)^{1/5}$$

d) $\frac{\log x}{7} - \frac{3}{5} \log y$

$$\log\left(\frac{x^{1/7}}{y^{3/5}}\right)$$

[8] 5. Expand the following logarithmic expressions (that is, write them using addition and subtraction of logarithms).

a) $\log_5(5xy)$

$$\log_5 5 + \log_5 x + \log_5 y$$

b) $\log_7(x^5 y^2)$

$$5 \log_7 x + 2 \log_7 y$$

c) $\log_8\left(\frac{x^{12}}{7}\right)$

$$12 \log_8 x - \log_8 7$$

d) $\log(x^4 y^3)^5$

$$5(4 \log x + 3 \log y)$$

[4] 6. Write the following logarithms in the indicated base.

a) $\log_5 7$, in base 2.

$$\log_5 7 = \frac{\log_2 7}{\log_2 5}$$

b) $\log_7 5$, in base 10.

$$\log_7 5 = \frac{\log 5}{\log 7}$$

[40] 7. Solve the following equations. If the answer is not an exact numbers, leave it expressed as a logarithm.

$$\text{a) } 7^{x+4} = 49 \rightarrow 7^{x+4} = 7^2 \Rightarrow x+4 = 2 \Rightarrow \boxed{x = -2}$$

Check:
 $\text{LHS} = 7^{(-2)+4} = 7^2 = 49 = \text{RHS} \checkmark$

$$\text{b) } 4^{x+3} = 8^{2x-4} \rightarrow 2^{2(x+3)} = 2^{3(2x-4)} \rightarrow 2(x+3) = 3(2x-4)$$

$$\rightarrow 2x+6 = 6x-12 \rightarrow -4x = -18 \rightarrow x = \frac{-18}{-4} = \boxed{\frac{9}{2}}$$

CHECK:
 $\text{LHS} = 4^{\frac{9}{2}+3} = 4^{\frac{15}{2}} = \boxed{2^{15}}$. $\text{RHS} = 8^{2 \cdot \frac{9}{2} - 4} = 8^5 = \boxed{2^{15}} \checkmark$

$$\text{c) } \log_2(x) - 3 = \log_2 5 \rightarrow \log_2(x) = 3 + \log_2 5 \rightarrow \log_2(x) - \log_2 5 = 3$$

$$\rightarrow \log_2 \frac{x}{5} = 3 \rightarrow \frac{x}{5} = 2^3 = 8 \rightarrow \boxed{x = 40}$$

Check:
 $\text{LHS} = \log_2(40) - 3 = \log_2 5 + \log_2 8 - 3 = \log_2 5 = \text{RHS} \checkmark$

$$\log_3(x-4)$$

$$\text{d) } \log_3(x-4) + \log_3(x-2) = \log_3(2x-7) \rightarrow \log_3((x-4)(x-2)) = \log_3(2x-7)$$

$$\rightarrow (x-4)(x-2) = 2x-7 \rightarrow x^2 - 6x + 8 = 2x - 7 \rightarrow x^2 - 8x + 15 = 0$$

$$\rightarrow (x-3)(x-5) = 0 \rightarrow \boxed{x = 3} \text{ or } \boxed{x = 5}$$

Check:

$$x = 3 \rightarrow \text{LHS} = \log_3(3-4) + \log_3(3-2) = \log_3(-1) + \log_3(1)$$

$x = 3$ is NOT a solution.

$$x = 5 \rightarrow \text{LHS} = \log_3(5-4) + \log_3(5-2) = 1 + 1 = 2$$

$$\text{RHS} = \log_3(2 \cdot 5 - 7) = \log_3 3 = 1 \checkmark$$

$$\boxed{x = 5}$$

[20] 8. For the rational function $f(x) = \frac{x^2 + 2x - 3}{x^2 - 2x - 3}$,

a) Factor numerator and denominator and simplify if possible.

$$f(x) = \frac{(x+3)(x-1)}{(x-3)(x+1)}$$

b) Find the x intercepts of the graph of $y = f(x)$, if they exist.

$$(x+3)(x-1) = 0 \Rightarrow x = -3 \text{ or } x = 1$$

c) Find the y intercepts of the graph of $y = f(x)$, if they exist.

$$f(0) = \frac{-3}{-3} = 1$$

d) Find any vertical asymptotes.

$$(x-3)(x+1) = 0 \rightarrow \text{V.A. at } x=3 \text{ and } x=-1$$

e) Find any horizontal asymptotes.

$$\text{As } x \rightarrow \pm\infty, f(x) \approx \frac{x^2}{x^2} = 1.$$

$$\Rightarrow \text{H.A. at } y = 1$$

f) Use the information above to sketch a graph of $y = f(x)$.

$$f(5) = \frac{(5+3)(5-1)}{(5-3)(5+1)}$$

$$= \frac{8 \cdot 4}{2 \cdot 6} = \frac{8}{3} \approx 2.6$$

