

MATH 30 - Precalculus, Version B

First Midterm. Time allowed: 2 hours, 15 minutes. Professor Luis Fernández

NAME: _____

[10] 1.

a) Fill in the blanks to complete the statement of the Remainder Theorem:

If the polynomial $f(x)$ is divided by $(x-a)$, then the remainder is $f(a)$.

b) What is the remainder when the polynomial $p(x) = x^{101} - 7x^{50} - 3x^9 - 8$ is divided by $(x+1)$?

$$\begin{aligned} p(-1) &= (-1)^{101} - 7(-1)^{50} - 3(-1)^9 - 8 \\ &= -1 - 7 + 3 - 8 = \boxed{-13} \end{aligned}$$

[10] 2.

a) Fill in the blanks to complete the statement of the Factor Theorem:

- If $f(a) = 0$, then $(x-a)$ is a factor of $f(x)$.
- If $(x-a)$ is a factor of $f(x)$, then $f(a) = 0$.

b) Find a polynomial of degree 4 with zeros at -2 , 3 , and -5 .

[NOTE: leave your polynomial factored; please do not expand it.]

$$\boxed{(x+2)(x-3)(x+5)^2}$$

- [10] 3. Divide using long division and write the answer as $D = d \cdot q + r$, where D is the dividend, d is the divisor, q is the quotient and r is the remainder.

$$\frac{6x^3 + 11x^2 - 2x - 12}{3x + 4}$$

$$\begin{array}{r}
 \boxed{2x^2 + x - 2} \\
 3x + 4 \overline{) 6x^3 + 11x^2 - 2x - 12} \\
 \underline{6x^3 + 8x^2} \\
 3x^2 - 2x \\
 \underline{3x^2 + 4x} \\
 -6x - 12 \\
 \underline{-6x - 8} \\
 \boxed{-4}
 \end{array}$$

$$6x^3 + 11x^2 - 2x - 12 = (3x + 4)(2x^2 + x - 2) - 4$$

- [10] 4. List all the possible rational roots of the polynomial $5x^6 - 14x^4 + 6x^2 - 9$.
NOTE: You are only asked to list them, NOT to factor the polynomial.

$$\text{Factors of } -9 \rightarrow 1, 3, 9$$

$$\text{Factors of } 5 \rightarrow 1, 5$$

$$\pm \left\{ 1, 3, 9, \frac{1}{5}, \frac{3}{5}, \frac{9}{5} \right\}$$

[12] 5.

a) Find the slope and equation of the line passing through the points (1, 2) and (3, -3),

$$m = \frac{-3 - 2}{3 - 1} = \frac{-5}{2}$$

Equation:

$$y - 2 = -\frac{5}{2}(x - 1)$$

or

$$y + 3 = -\frac{5}{2}(x - 3)$$

b) Find the equation of the line perpendicular to the line $y = \frac{2x}{3} + 4$ and passing through the point (1, 2).

Slope of given line: $m_1 = \frac{2}{3}$

Slope of perpendicular: $m_2 = -\frac{3}{2}$.

Equation of perpendicular through (1, 2):

$$y - 2 = -\frac{3}{2}(x - 1)$$

[12] 6. Consider the line given by the equation $2x + 3y = 6$.

a) Find its slope and y -intercept.

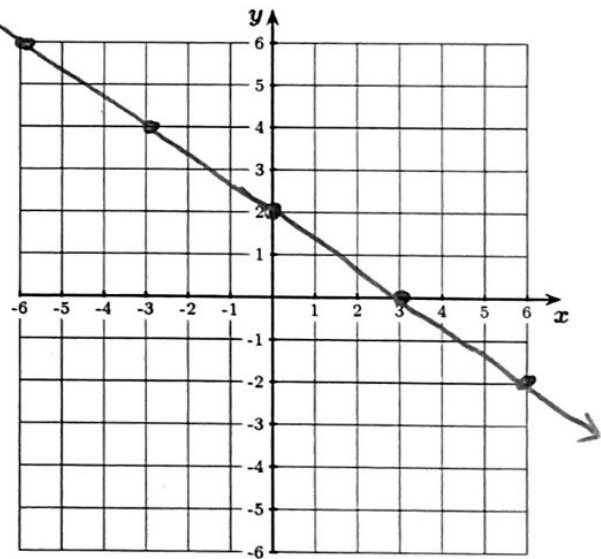
Solve for y :

$$2x + 3y = 6 \Rightarrow 3y = -2x + 6$$

$$\Rightarrow y = -\frac{2}{3}x + 2$$

$\text{Slope} = -\frac{2}{3}$ $y\text{-intercept} = 2$
--

b) Graph the line in the coordinate axes below.



- [12] 7. For the quadratic function $f(x) = -2(x-1)^2 + 2$,

a) Find the vertex.

$$\begin{array}{c} h \quad k \\ \downarrow \quad \downarrow \\ (1, 2) \end{array}$$

b) Find the x -intercepts, if any.

$$\begin{aligned} -2(x-1)^2 + 2 &= 0 \Rightarrow \frac{-2(x-1)^2}{-2} = \frac{-2}{-2} \Rightarrow (x-1)^2 = 1 \\ \Rightarrow (x-1) &= \pm 1 \Rightarrow x = 1 \pm 1 \end{aligned}$$

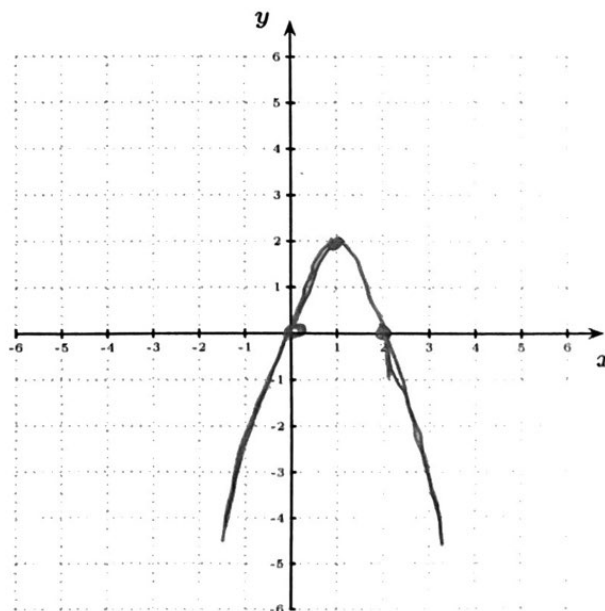
$$\boxed{x\text{-intercepts: } 0 \text{ and } 2}$$

c) Find the y -intercepts.

$$f(0) = -2(0-1)^2 + 2 = \frac{-2(-1)^2}{-2} + 2 = -2 + 2 = \boxed{0}$$

d) Determine whether the parabola opens up or down.
Sketch the graph on the coordinate axes provided.

Opens down
because $a = -2 < 0$.



[12] 8. Find all the solutions of the equation $x^3 - 5x^2 + 5x - 1 = 0$

[NOTE: one of the solutions is rational, so it can be found using synthetic division. The other two are irrational; to find them you need to use the quadratic formula or complete the square.]

Start using synthetic division.

Candidates: $\frac{\text{Factors of } -1 \rightarrow \pm 1}{\text{Factors of } 1 \rightarrow \pm 1}$

Candidates: 1, -1.

Try 1:

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 5 & -1 \\ & & 1 & -4 & 1 \\ \hline & 1 & -4 & 1 & 0 \end{array} \rightarrow (x-1)$$

\rightarrow Solve $(x-1)(x^2 - 4x + 1) = 0$

$$\begin{array}{l} \downarrow \\ x-1=0 \\ \boxed{x=1} \end{array}$$

\downarrow
 $x^2 - 4x + 1 = 0$
Use quadratic formula.
 $a=1, b=-4, c=1.$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16-4}}{2}$$

$$= \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = \boxed{2 \pm \sqrt{3}}$$

Solutions:

$$\boxed{x=1, x=2+\sqrt{3}, x=2-\sqrt{3}}$$

[12] 9. Factor completely the polynomial $f(x) = x^4 + 4x^3 - 6x^2 - 4x + 5$.

Use synthetic division.

Candidates: $\frac{\text{Factors of } 5}{\text{Factors of } 1} \rightarrow 1, 5$
 $\rightarrow 1$

Candidates: $\{+1, +5, -1, -5\}$.

Try 1:
$$\begin{array}{r|rrrrr} 1 & 1 & 4 & -6 & -4 & 5 \\ & & \downarrow & & & \\ & & 1 & 5 & -1 & -5 \\ \hline & & & & & 0 \end{array} \rightarrow (x-1)$$

Try 1:
$$\begin{array}{r|rrrr} 1 & 1 & 6 & 5 \\ & & \downarrow & & \\ & & 1 & 6 & 5 \\ \hline & & & & 0 \end{array} \rightarrow (x-1)$$

Try 1:
$$\begin{array}{r|rr} 1 & 1 & 7 \\ & & \downarrow \\ & & 1 & 7 \\ \hline & & & 0 \end{array} \text{ Yes!}$$

Try 1:
$$\begin{array}{r|rrrr} 1 & 1 & 7 & 12 \\ & & \downarrow & & \\ & & 1 & 7 & 12 \\ \hline & & & & 0 \end{array} \text{ NO X}$$


Try -1:
$$\begin{array}{r|rrr} -1 & 1 & 6 & 5 \\ & & \downarrow & \\ & & -1 & -5 \\ \hline & & 1 & 5 \\ \hline & & & 0 \end{array} \rightarrow (x+1)$$

 $(x+5)$

$f(x) = (x-1)^2(x+1)(x+5)$

[12] 10. The polynomial $f(x) = x^3 - 3x - 2$ can be factored as $f(x) = (x + 1)^2(x - 2)$.





a) Find the end behavior of f .

Degree 3, $a = 1 > 0 \Rightarrow$ 

b) Find the x -intercepts of f and their multiplicity, and the local behavior at the intercepts.

$$f(x) = 0 \Rightarrow (x+1)^2(x-2) = 0$$

\downarrow \downarrow
 -1 2

$x = -1$ with multiplicity 2		or	
$x = 2$ " " 1		or	

c) Find the y -intercept of f .

$f(0) = -2$

d) Sketch the graph of f in the axes provided.

