MATH 30 - Precalculus, Version A

First Midterm. Time allowed: 2 hours, 15 minutes.   Professor Luis Fernández

NAME: _____ SOLUTION _____

[10] **1.**

   a) Fill in the blanks to complete the statement of the Remainder Theorem:

   If the polynomial $f(x)$ is divided by $(x - a)$, then the remainder is $f(a)$

   b) What is the remainder when the polynomial $p(x) = x^{100} + 5x^{50} - 6x^{23} + 5$ is divided by $(x + 1)$?

$$p(-1) = (-1)^{100} + 5(-1)^{50} - 6(-1)^{23} + 5$$
$$= 1 + 5 + 6 + 5 = \boxed{17}$$

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[10] **2.**

   a) Fill in the blanks to complete the statement of the Factor Theorem:

   • If $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$.

   • If $(x - a)$ is a factor of $f(x)$, then $f(a) = 0$

   b) Find a polynomial **of degree 4** with zeros at -4, 5, and -6.
   [NOTE: leave your polynomial factored; please do not expand it.]

$$\boxed{(x+4)(x-5)(x+6)^2}$$

[10] **3.** Divide using long division and write the answer as $D = d \cdot q + r$, where D is the dividend, d is the divisor, q is the quotient and r is the remainder.

$$\frac{6x^3 - 6x^2 - 24x - 11}{2x + 2}$$

$$
\begin{array}{r}
3x^2 - 6x - 6 \\
2x+2 \overline{\smash{\big)}\, 6x^3 - 6x^2 - 24x - 11} \\
-\underline{6x^3 + 6x^2} \\
-12x^2 - 24x \\
-\underline{-12x^2 - 12x} \\
-12x - 11 \\
-\underline{-12x - 12} \\
1
\end{array}
$$

$$6x^3 - 2x^2 - 24x - 11 = (2x+2)(3x^2 - 6x - 6) + 1$$

[10] **4.** **List** all the possible rational roots of the polynomial $3x^6 - 3x^2 - 15x + 4$.

NOTE: You are only asked to list them, NOT to factor the polynomial.

$$\frac{p}{q} \quad \text{where } p \text{ is a factor of } 4 \rightarrow 1, 2, 4$$

$$\phantom{\frac{p}{q}} \quad\quad\quad\quad q \quad \text{"} \quad \text{"} \quad \text{"} \quad \text{"} \quad 3 \rightarrow 1, 3$$

$$\Rightarrow \quad \pm \left\{ 1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3} \right\}$$

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[12] 5.

a) Find the slope and equation of the line passing through the points $(1, 2)$ and $(3, -3)$,

$$m = \frac{-3 - 2}{3 - 1} = -\frac{5}{2}$$

Equation: $\boxed{y - 2 = -\frac{5}{2}(x - 1)}$

or

$\boxed{y + 3 = -\frac{5}{2}(x - 3)}$

b) Find the equation of the line perpendicular to the line $y = \frac{2x}{3} + 4$ and passing through the point $(1, 2)$.

Slope of given line: $m_1 = \frac{2}{3}$

Slope of perpendicular: $m_2 = -\frac{3}{2}$.

Equation of perpendicular line passing through $(1, 2)$:

$\boxed{y - 2 = -\frac{3}{2}(x - 1)}$

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[12] **6.** Consider the line given by the equation $2x + 3y = 6$.

a) Find its slope and y-intercept.

Solve for y: $2x + 3y = 6$

$$3y = -2x + 6$$

$$y = -\frac{2}{3}x + 2$$

slope: $-\frac{2}{3}$

y-intercept: 2

b) Graph the line in the coordinate axes below.

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$$\begin{array}{cc} h & k \\ \downarrow & \downarrow \end{array}$$
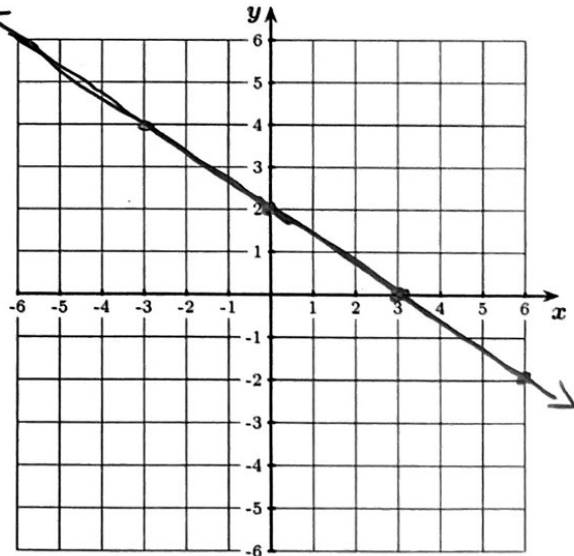
[12] **7.** For the quadratic function $f(x) = -2(x-2)^2 + 2$,

a) Find the vertex.

$$\boxed{\begin{array}{c} h \quad k \\ (2, 2) \end{array}}$$

b) Find the x-intercepts, if any.

$$-2(x-2)^2 + 2 = 0 \implies \frac{-2(x-2)^2}{-2} = \frac{-2}{-2}$$

$$\implies (x-2)^2 = 1$$

$$\implies x - 2 = \pm 1 \implies x = 2 \pm 1 \begin{array}{c} \nearrow 3 \\ \searrow 1 \end{array}$$

x intercepts are 1 and 3.

c) Find the y-intercepts.

$$f(0) = -2((0)-2)^2 + 2 = -2 \cdot (-2)^2 + 2$$

$$= -2(4) + 2 = -8 + 2$$

$$= \boxed{-6}$$

d) Determine whether the parabola opens up or down. Sketch the graph on the coordinate axes provided.

Opens down since $a = -2 < 0$

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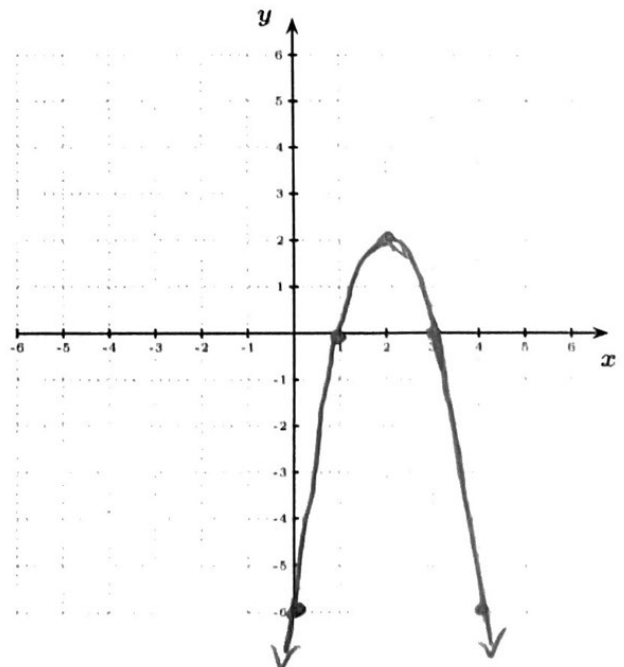
[12] **8.** Find all the solutions of the equation $x^3 - 5x^2 + 6x - 2 = 0$.

[NOTE: one of the solutions is rational, so it can be found using synthetic division. The other two are irrational; to find them you need to use the quadratic formula or complete the square.]

Use synthetic division to factor $(x^3 - 5x^2 + 6x - 2)$

Candidates: $\dfrac{\text{Factors of } (-2)}{\text{Factors of } 1}$

$\Rightarrow 1, -1, 2, -2.$

Try 1:

$$\begin{array}{c|ccc} 1 & 1 & -5 & 6 & -2 \\ & & 1 & -4 & 2 \\ \hline & 1 & -4 & 2 & \boxed{0} \end{array}$$

$\Rightarrow$ Solve $(x-1)(x^2 - 4x + 2) = 0$

$\Rightarrow x - 1 = 0$ $x^2 - 4x + 2 = 0$

$\boxed{x = 1}$ Use quadratic formula:

$a = 1$
$b = -4$
$c = 2$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 8}}{2}$$

$$= \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}.$$

Solutions:
$x = 1, \quad x = 2 + \sqrt{2}, \quad x = 2 - \sqrt{2}$

[12] **9.** Factor completely the polynomial $f(x) = x^4 + 2x^3 - 4x^2 - 2x + 3$.

Candidates: $\dfrac{\text{Factors of } 3}{\text{Factors of } 1} \rightarrow \dfrac{1, 3}{1}$

$\Rightarrow$ Candidates: $1, 3, -1, -3$

Try 1:

	1	2	-4	-2	3
1	↓	1	3	-1	-3
	1	3	-1	-3	0 ✓
1	↓	1	4	3	
	1	4	3	0 ✓	
1	↓	1	5		
	1	5	8 NO ✗		

$\rightarrow (x-1)$

$\rightarrow (x-1)$ Yes

Try -1:

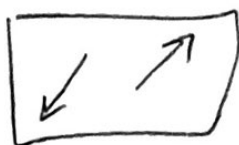|	1	4	3
-1	↓	-1	-3
	1	3	0 Yes ✓

$\rightarrow (x+1)$

$(x+3)$

$$\boxed{f(x) = (x-1)^2 (x+1)(x+3)}$$

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[12] **10.** The polynomial $f(x) = x^3 - 3x + 2$ can be factored as $f(x) = (x-1)^2(x+2)$.

a) Find the end behavior of f.

$a = 1$, degree 3 $\rightarrow$

b) Find the x-intercepts of f and their multiplicity, and the local behavior at the intercepts.

x-intercepts: $(x-1)^2(x+2) = 0$

y_{+1} y_{-2}

1 with multiplicity 2 $\rightarrow$ $\smile$ or $\frown$

-2 "   "   1 $\rightarrow$ $\diagup$ ~ $\diagdown$

c) Find the y-intercept of f.

$f(0) = 2$

d) Sketch the graph of f in the axes provided.

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