MATH 30 - Precalculus, Version B

First Midterm. Time allowed: 2 hours, 15 minutes. Professor Luis Fernández

NAME: SOLUTION

[6] **1.** Carefully write down the statement of the Remainder Theorem:

If $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$

[6] **2.** Suppose that we divide the polynomial $p(x) = x^{101} - 7x^{50} - 3x^9 - 8$ by $(x+1)$. What remainder do we get?

$$p(-1) = (-1)^{101} - 7(-1)^{50} - 3(-1)^9 - 8$$
$$= -1 - 7 + 3 - 8 = \boxed{-13}$$

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[10] **3.** Divide using long division and write the answer as $D = d \cdot q + r$, where D is the dividend, d is the divisor, q is the quotient and r is the remainder.

$$\frac{6x^3 + 11x^2 - 2x - 12}{3x+4}$$

$$\begin{array}{r|l} & 2x^2 + x - 2 \\ \hline 3x+4 & 6x^3 + 11x^2 - 2x - 12 \\ & -6x^3 - 8x^2 \\ \hline & 3x^2 - 2x \\ & -3x^2 - 4x \\ \hline & -6x - 12 \\ & +6x + 8 \\ \hline & -4 \end{array}$$

$$\boxed{6x^3 + 11x^2 - 2x - 12 = (3x+4)(2x^2 + x - 2) - 4}$$

[10] **4.** **List** all the possible rational roots of the polynomial $5x^6 - 14x^4 + 6x^2 - 9$.
NOTE: You are only asked to list them, NOT to factor the polynomial.

They have the form $\frac{p}{q}$ where

- p is a factor of 9 $\rightarrow 1, 3, 9$
- q " " " " 5 $\rightarrow 1, 5$.

$$\boxed{\text{Possible zeros: } \pm\left(1, 3, 9, \frac{1}{5}, \frac{3}{5}, \frac{9}{5}\right)}$$

[12] **5.** Find the equation of the line perpendicular to the line $y = \dfrac{2x}{3} + 4$ and passing through the point $(1, 2)$.

The slope of $y = \frac{2x}{3} + 4$ is $\frac{2}{3}$.

The slope of the perpendicular is $-\frac{3}{2}$.

Using the point slope form, the equation we want is

$$\boxed{y - 2 = -\frac{3}{2}(x-1)}$$

$$\left(\text{or } y = -\frac{3}{2}x + \frac{7}{2}\right)$$

[12] **6.** Consider the line given by the equation $2x + 3y = 6$.

a) Find its slope and y-intercept.

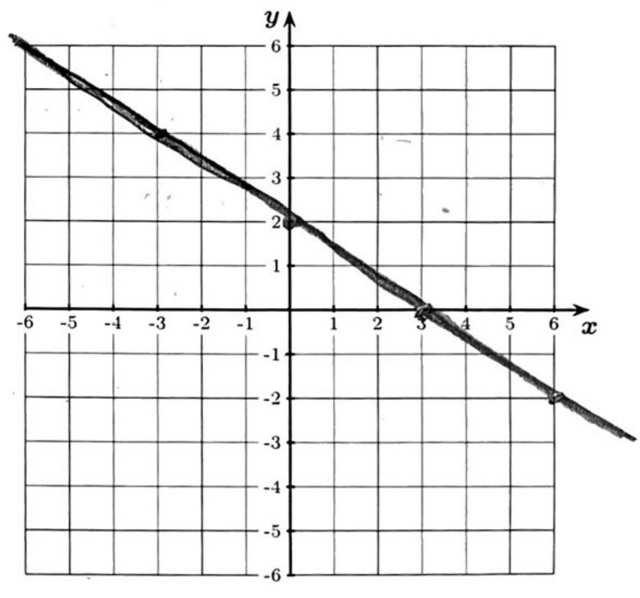
Solve for y: $2x+3y=6$

$\Rightarrow 3y = -2x+6$

$\Rightarrow y = -\frac{2}{3}x + 2$

$\boxed{\text{slope: } -\frac{2}{3} \quad y\text{-intercept: } 2}$

b) Graph the line in the coordinate axes below.



[12] **7.** For the quadratic function $f(x) = -2(x-1)^2 + 2$,

a) Find the vertex.

$\boxed{\text{Vertex: }(1, 2)}$

b) Find the x-intercepts, if any.

Solve $-2(x-1)^2 + 2 = 0$

$\quad\quad\quad\quad -2 \quad -2$

$\Rightarrow \dfrac{-2(x-1)^2}{-2} = \dfrac{-2}{-2}$

$(x-1)^2 = 1$

$x - 1 = \pm 1$

$x = 1 \pm 1$

$\boxed{x = 2 \quad x = 0}$

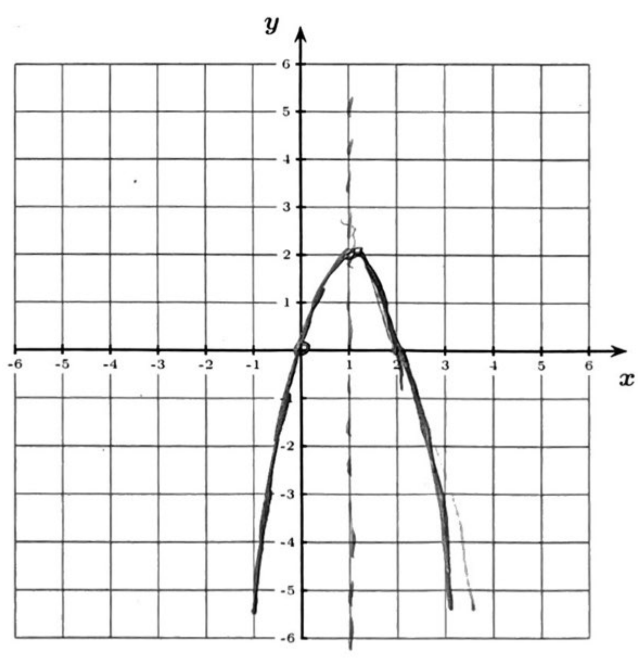
c) Find the y-intercepts.

$f(0) = -2(0-1)^2 + 2 = -2\overbrace{(-1)^2}^{1} + 2 = -2 + 2 = \boxed{0}$

d) Determine whether the parabola opens up or down. Sketch the graph on the coordinate axes provided.

Opens down

$(-2 < 0)$



[12] **8.** Find all the solutions of the equation $x^3 - 5x^2 + 5x - 1 = 0$.
[NOTE: one of the solutions is rational, so it can be found using synthetic division. The other two are irrational; to find them you need to use the quadratic formula or complete the square.]

Possible zeros: $\frac{p}{q}$, with p factor of $1 \to 1$
q " " $1 \to 1$.

$\boxed{\text{Possible zeros are } 1, -1}$

Try 1:

$$\begin{array}{r|rrrr} & 1 & -5 & 5 & -1 \\ 1 & & +1 & -4 & 1 \\ \hline & 1 & -4 & 1 & \boxed{0} \end{array}$$

Works! $\to$ get $(x-1)(x^2-4x+1)=0$

Try 1 again

$$\begin{array}{r|rrr} 1 & & 1 & -3 \\ \hline & 1 & -3 & \boxed{-2} \end{array}$$

NO

Try -1:

$$\begin{array}{r|rrr} & 1 & -4 & 1 \\ -1 & & -1 & 5 \\ \hline & 1 & -5 & \boxed{6} \end{array}$$

NO.

So we need to solve

$$(x-1)(x^2-4x+1)=0$$

$$\underbrace{(-4)^2}_{16}-4(1)(1) = 16-4 = 12$$

$$\sqrt{12} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$$

$x - 1 = 0$

$\boxed{x = 1}$

$x^2 - 4x + 1 = 0$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{4 \pm 2\sqrt{3}}{2} = \frac{4}{2} \pm \frac{2\sqrt{3}}{2}$$

$$= \boxed{2 \pm \sqrt{3}}$$

Solutions:

$\boxed{x = 1, \quad x = 2+\sqrt{3}, \quad x = 2-\sqrt{3}}$

[12] **9.** Factor completely the polynomial $f(x) = x^4 + 4x^3 - 6x^2 - 4x + 5$.

Possible zeros: $\frac{p}{q}$ with p factor of $5 \to 1, 5$
q " " " $1 \to 1$.

Possible zeros: $1, 5, -1, -5$.

Try 1:

$$\begin{array}{r|rrrrr} & 1 & 4 & -6 & -4 & 5 \\ 1 & \downarrow & 1 & 5 & -1 & -5 \\ \hline & 1 & 5 & -1 & -5 & \boxed{0} \end{array} \quad \text{Yes!} \to \boxed{(x-1)}$$

Try 1:

$$\begin{array}{r|rrrr} 1 & & 1 & 6 & 5 \\ \hline & 1 & 6 & 5 & \boxed{0} \end{array} \quad \text{Yes!} \to \boxed{(x-1)}$$

Try 1:

$$\begin{array}{r|rrr} 1 & & 1 & 7 \\ \hline & 1 & 7 & \boxed{12} \end{array} \quad \text{NO}.$$

Try -1:

$$\begin{array}{r|rrr} & 1 & 6 & 5 \\ -1 & & -1 & -5 \\ \hline & 1 & 5 & \boxed{0} \end{array} \quad \text{Yes!} \to \boxed{(x+1)}$$

$\boxed{(x+5)}$.

Solution:

$$\boxed{x^4 + 4x^3 - 6x^2 - 4x + 5 = (x-1)^2(x+1)(x+5)}$$

[12] **10.** The polynomial $f(x) = x^3 - 3x - 2$ can be factored as $f(x) = (x+1)^2(x-2)$.

a) Find the end behavior of f.

Degree 3, leading coefficient 1

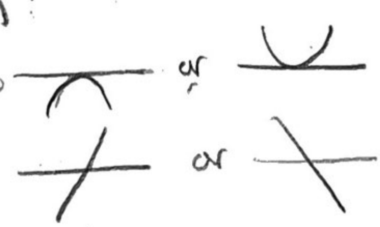
$\Rightarrow$ ↙ ↗

b) Find the x-intercepts of f and their multiplicity, and the local behavior at the intercepts.

x-intercepts of $f(x) = (x+1)^2(x-2)$:

-1 with multiplicity 2 →

2 " " 1 →



c) Find the y-intercept of f.

$\boxed{f(0) = -2}$

d) Sketch the graph of f in the axes provided.