MATH 30 - Precalculus, Version A

First Midterm. Time allowed: 1 hour, 25 minutes. Professor Luis Fernández

NAME: SOLUTION

- **Show your work** in order to get credit. Scientific calculators allowed. Graphing calculators not allowed.
- The midterm has 8 questions, and 104 points, including 4 extra credit points.
- Write your answers in the space provided.

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[6] **1.** Carefully write down the statement of the Remainder Theorem:

The value of $f(a)$ is the same as the remainder of $f(x) \div (x-a)$

[6] **2.** Suppose that we divide the polynomial $p(x) = x^{100} + 5x^{50} - 6x^{23} + 5$ by $(x+1)$. What remainder do we get?

$(x-(-1))$

The remainder is equal to the value of $p(-1)$.

$$\begin{aligned} p(-1) &= (-1)^{100} + 5(-1)^{50} - 6(-1)^{23} + 5 \\ &= 1 + 5(1) - 6(-1) + 5 \\ &= 1+5+6+5 = \boxed{17} \end{aligned}$$

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[10] **3.** Divide using long division and write the answer as $D = d \cdot q + r$, where D is the dividend, d is the divisor, q is the quotient and r is the remainder.

$$\frac{6x^3 - 8x^2 - 24x - 11}{2x+2}$$

$$\begin{array}{r|l} & 3x^2 - 7x - 5 \\ \hline 2x+2 & 6x^3 - 8x^2 - 24x - 11 \\ & \underline{-6x^3 - 6x^2} \\ & \quad -14x^2 - 24x \\ & \quad \underline{+14x^2 + 14x} \\ & \qquad -10x - 11 \\ & \qquad \underline{+10x + 10} \\ & \qquad\qquad -1 \end{array}$$

$$\boxed{3x^2 - 7x - 5 = (2x+2)(3x^2-7x-5) - 1}$$

[10] **4.** **List** all the possible rational roots of the polynomial $3x^6 - 3x^2 - 15x + 4$.
NOTE: You are only asked to list them, NOT to factor the polynomial.

They have the form $\frac{p}{q}$ where p is a factor of 4
q " " " " 3.

$\Rightarrow$ $p = 1, 2, 4$
$q = 1, 3$ $\Rightarrow$ $\boxed{\pm\left\{1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}\right\}}$

[12] **5.** Find the equation of the line perpendicular to the line $y = \frac{2x}{3} + 4$ and passing through the point $(1, 2)$.

The slope of the given line is $\frac{2}{3}$.
The slope of the perpendicular is $-\frac{3}{2}$.
Therefore, the equation we want is

$$\boxed{y - 2 = -\frac{3}{2}(x-1)}$$

(or $y = 2 - \frac{3x}{2} + \frac{3}{2} = -\frac{3x}{2} + \frac{7}{2}$)

or $\boxed{y = -\frac{3x}{2} + \frac{7}{2}}$

[20] **6.** For the quadratic function $f(x) = -2(x-2)^2 + 4$,

a) Find the vertex.

$\boxed{(2,4)}$

b) Find the x-intercepts, if any.

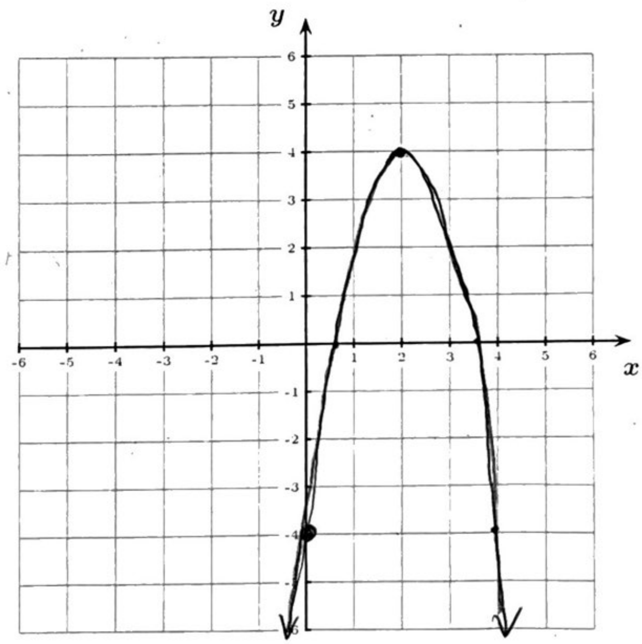
$$-2(x-2)^2+4=0 \implies -2(x-2)^2=-4$$
$$\implies (x-2)^2 = \frac{-4}{-2} = 2$$
$$\implies (x-2) = \pm\sqrt{2}$$
$$\implies x = 2 \pm \underbrace{\boxed{\sqrt{2}}}_{\approx 1.4}$$
$$\implies \boxed{\begin{array}{l} x = 2+\sqrt{2} \approx 3.4 \\ x = 2-\sqrt{2} \approx 0.6 \end{array}}$$

c) Find the y-intercepts.

$$f(0) = -2(0-2)^2+4 = -2(4)+4 = -8+4 = \boxed{-4}$$

d) Determine whether the parabola opens up or down. Sketch the graph on the coordinate axes provided.

Opens down
because $a=-2<0$



[20] **7.** Find all the solutions of the equation $x^4 - 9x^3 + 17x^2 - 3x - 6 = 0$.
[NOTE: two of the solutions are rational, so they can be found using synthetic division. The other two are irrational; to find them you need to use the quadratic formula or complete the square.]

The possible rational solutions are

$\frac{p}{q}$ where p is a factor of $6 \to 1, 2, 3, 6$

$\phantom{\frac{p}{q}}$ " q " " " " $1 \to 1$

$\Rightarrow$ Possible roots are $\pm\{1, 2, 3, 6\}$.

Try 1:

$$\begin{array}{r|rrrrr} & 1 & -9 & 17 & -3 & -6 \\ 1 & & 1 & -8 & 9 & 6 \\ \hline & 1 & -8 & 9 & 6 & \boxed{0} \end{array} \text{ Yes!} \to \boxed{(x-1)}$$

1 again

$$\begin{array}{r|rrrr} 1 & \downarrow & 1 & -7 & 2 \\ \hline & 1 & -7 & 2 & \boxed{8} \end{array} \text{ NO}$$

Try -1:

$$\begin{array}{r|rrrr} & 1 & -8 & 9 & 6 \\ -1 & & -1 & 9 & -18 \\ \hline & 1 & -9 & 18 & \boxed{-12} \end{array} \text{ NO}$$

Try 2:

$$\begin{array}{r|rrrr} & 1 & -8 & 9 & 6 \\ 2 & \downarrow & 2 & -12 & -6 \\ \hline & \boxed{1 \quad -6 \quad -3} & & & \boxed{0} \end{array} \text{ Yes}$$

$\to \boxed{(x-2)}$

$x^2 - 6x - 3$.

We get the equation

$$(x-1)(x-2)(x^2-6x-3) = 0$$

$x - 1 = 0 \Rightarrow \boxed{x = 1}$ $\qquad$ $x - 2 = 0 \Rightarrow \boxed{x = 2}$ $\qquad$ $x^2 - 6x - 3 = 0$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-3)}}{2(1)}$$

$36 + 12 = 48$

$48 = 16 \cdot 3$.

$$= \frac{6 \pm \sqrt{48}}{2} = \frac{6 \pm 4\sqrt{3}}{2}$$

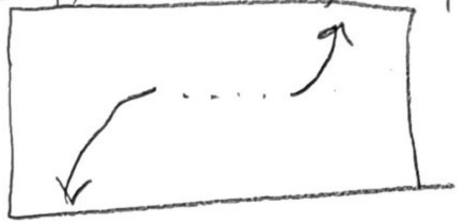
$$= \boxed{3 \pm 2\sqrt{3}}$$

$\boxed{\text{The solutions are } 1, 2, 3+2\sqrt{3}, 3-2\sqrt{3}}$

[20] **8.** The polynomial $f(x) = x^3 - 3x + 2$ can be factored as $f(x) = (x-1)^2(x+2)$.

a) Find the end behavior of f.

Leading coefficient is 1, degree is 3 (odd):



b) Find the x-intercepts of f and their multiplicity, and the local behavior at the intercepts.

$(x-1)^2(x+2) = 0$

$\Rightarrow (x-1)^2 = 0$ or $(x+2) = 0$

$\Downarrow$ $\qquad\qquad$ $\Downarrow$

$x - 1 = 0$

$\boxed{x = 1}$ $\qquad$ $\boxed{x = -2}$

The x-intercepts are

$x = 1$, multiplicity 2

$x = -2$, " 1

c) Find the y-intercept of f.

$f(0) = (0)^3 - 3(0) + 2 = \boxed{2}$

d) Sketch the graph of f in the axes provided.