MATH 30 - Precalculus, Version A

First Midterm. Time allowed: 2 hours, 15 minutes. Professor Luis Fernández

NAME: ______________________________

[6] **1.** Carefully write down the statement of the Remainder Theorem:

If $f(x)$ is divided by $(x-a)$, the remainder is $f(a)$

[6] **2.** Suppose that we divide the polynomial $p(x) = x^{100} + 5x^{50} - 6x^{23} + 5$ by $(x+1)$. What remainder do we get?

$$p(-1) = (-1)^{100} + 5(-1)^{50} - 6(-1)^{23} + 5$$
$$= 1 + 5 + 6 + 5 = \boxed{16}$$

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[10] **3.** Divide using long division and write the answer as $D = d \cdot q + r$, where D is the dividend, d is the divisor, q is the quotient and r is the remainder.

$$\frac{6x^3 - 8x^2 - 24x - 11}{2x+2}$$

$$\begin{array}{r|l}
 & 3x^2 - 7x - 5 \\
\hline
2x+2 & 6x^3 - 8x^2 - 24x - 11 \\
 & -(6x^3 + 6x^2) \\
\hline
 & -14x^2 - 24x \\
 & -(-14x^2 - 14x) \\
\hline
 & -10x - 11 \\
 & -(-10x - 10) \\
\hline
 & -1
\end{array}$$

$$\boxed{6x^3 - 8x^2 - 24x - 11 = (2x+2)(3x^2 - 7x - 5) - 1}$$

[10] **4.** **List** all the possible rational roots of the polynomial $3x^6 - 3x^2 - 15x + 4$.
NOTE: You are only asked to list them, NOT to factor the polynomial.

They are of the form $\frac{p}{q}$ with p factor of $4 \to 1, 2, 4$.
q " " $3 \to 1, 3$.

Therefore, the possible rational roots are

$$\boxed{\pm\left(1, 2, 4, \frac{1}{3}, \frac{2}{3}, \frac{4}{3}\right)}$$

[12] **5.** Find the equation of the line perpendicular to the line $y = \dfrac{2x}{3} + 4$ and passing through the point $(1, 2)$.

The slope of $y = \frac{2x}{3} + 4$ is $\frac{2}{3}$.

The slope of the perpendicular is $-\frac{3}{2}$.

Using the point-slope form, the equation we want is

$$\boxed{y - 2 = -\frac{3}{2}(x - 1)}$$

$$\left(\text{or } y = -\frac{3}{2}x + \frac{7}{2}\right)$$

[12] **6.** Consider the line given by the equation $2x + 3y = 6$.

a) Find its slope and y-intercept.

Solve for y: $2x + 3y = 6$

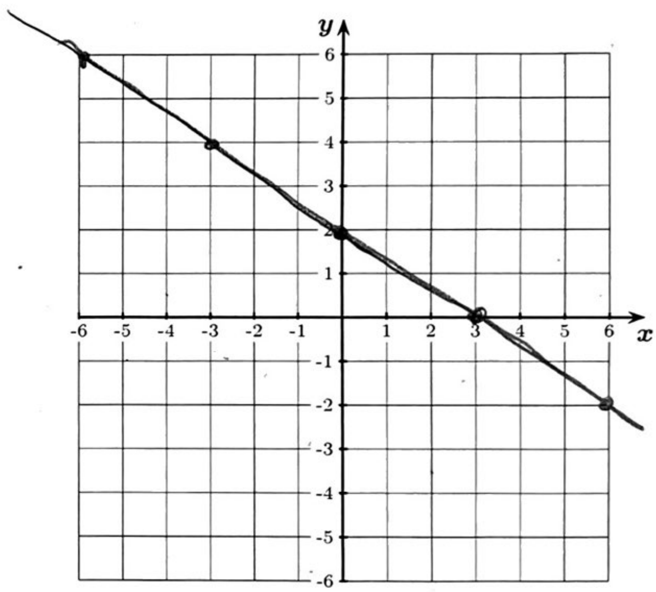
$-2x \qquad -2x$

$3y = -2x + 6$

$y = -\frac{2}{3}x + 2$

$\boxed{\text{Slope: } -\frac{2}{3} \quad \text{y-intercept: } 2}$

b) Graph the line in the coordinate axes below.



[12] **7.** For the quadratic function $f(x) = -2(x-2)^2 + 4$,

a) Find the vertex.

$\boxed{\text{Vertex: } (2,4)}$

b) Find the x-intercepts, if any.

Solve $-2(x-2)^2 + 4 = 0$

$\qquad\qquad -4 \quad -4$

$\Rightarrow -2(x-2)^2 = -4$

$(x-2)^2 = 2$

$x - 2 = \pm\sqrt{2}$

$\boxed{x = 2 \pm \sqrt{2}}$

$x = 2 + \sqrt{2} \approx 3.4$

$x = 2 - \sqrt{2} \approx 0.6$

c) Find the y-intercepts.

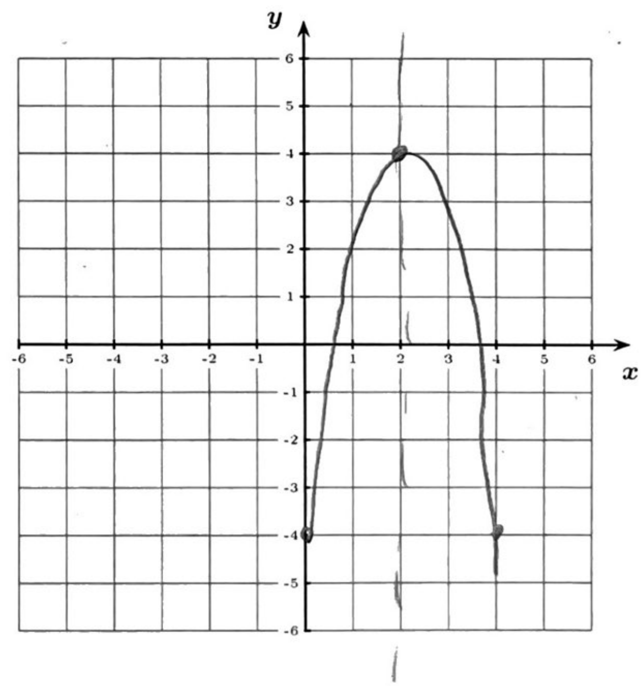
$f(0) = -2(0-2)^2 + 4 = -2(2)^2 + 4$

$= -8 + 4$

$= -4$

d) Determine whether the parabola opens up or down. Sketch the graph on the coordinate axes provided.

Opens down



[12] **8.** Find all the solutions of the equation $x^3 - 5x^2 + 6x - 2 = 0$.
[NOTE: one of the solutions is rational, so it can be found using synthetic division. The other two are irrational; to find them you need to use the quadratic formula or complete the square.]

Possible zeros: $\frac{p}{q}$ with p factor of $2 \to 1, 2$
q " " $1 \to 1$.

Possible zeros: $1, -1, 2, -2$.

Try 1:

$$\begin{array}{r|rrrr} & 1 & -5 & 6 & -2 \\ 1 & & 1 & -4 & 2 \\ \hline & 1 & -4 & 2 & \boxed{0} \end{array}$$

Works. $\to (x-1)(x^2-4x+2)=0$

Try 1

$$\begin{array}{r|rrr} 1 & & 1 & -3 \\ \hline & 1 & -3 & +1 \end{array}$$

NO

Try 2:

$$\begin{array}{r|rrr} & 1 & -4 & 2 \\ 2 & & 2 & -4 \\ \hline & 1 & -2 & +2 \end{array}$$

NO

Try (-2)

$$\begin{array}{r|rrr} & 1 & -4 & 2 \\ -2 & & -2 & 12 \\ \hline & 1 & -6 & \boxed{14} \end{array}$$

NO.

So we need to solve

$$(x-1)(x^2-4x+2)=0$$

$x-1=0$ or $x^2-4x+2=0$

$\boxed{x=1}$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2-4(1)(2)}}{2(1)}$$

$(-4)^2-4(1)(2) = 16-8 = 8$.

$\sqrt{8} = \sqrt{4}\sqrt{2} = 2\sqrt{2}$

$$= \frac{4 \pm 2\sqrt{2}}{2} = \frac{4}{2} \pm \frac{2\sqrt{2}}{2} = 2 \pm \sqrt{2}.$$

Solutions:
$1,\ 2+\sqrt{2},\ 2-\sqrt{2}$

[12] **9.** Factor completely the polynomial $f(x) = x^4 + 2x^3 - 4x^2 - 2x + 3$.

Possible zeros: $\frac{p}{q}$ with p factor of 3 $\to 1, 3$

q " " $1 \to 1$

Possible zeros: $1, -1, 3, -3$.

Try 1

$$\begin{array}{r|rrrrr} & 1 & 2 & -4 & -2 & 3 \\ 1 & \downarrow & 1 & 3 & -1 & -3 \\ \hline & 1 & 3 & -1 & -3 & \boxed{0} \end{array} \quad \text{Yes!} \quad \to \boxed{(x-1)}$$

Try 1

$$\begin{array}{r|rrrr} 1 & \downarrow & 1 & 4 & 3 \\ \hline & 1 & 4 & 3 & \boxed{0} \end{array} \quad \text{Yes!} \quad \to (x-1)$$

Try 1 $\to$ No:

$$\begin{array}{r|rrr} 1 & & 1 & 5 \\ \hline & 1 & 5 & \boxed{8} \end{array} \quad \underline{\text{NO}}$$

Try -1

$$\begin{array}{r|rrr} & 1 & 4 & 3 \\ -1 & & -1 & -3 \\ \hline & 1 & 3 & \boxed{0} \end{array} \quad \text{Yes} \quad \to (x+1)$$

$(x+3)$

Solution:

$$\boxed{x^4 + 2x^3 - 4x^2 - 2x + 3 = (x-1)^2(x+1)(x+3)}$$

[12] **10.** The polynomial $f(x) = x^3 - 3x + 2$ can be factored as $f(x) = (x-1)^2(x+2)$.

a) Find the end behavior of f.

Degree = 3, leading coefficient = 1.

$\boxed{\swarrow \quad \nearrow}$

b) Find the x-intercepts of f and their multiplicity, and the local behavior at the intercepts.

x = 1, with multiplicity 2 → or

x = −2, " " 1 → or

c) Find the y-intercept of f.

$\boxed{f(0) = 2}$

d) Sketch the graph of f in the axes provided.