MATH 30 - Precalculus. Review exercises. Professor Luis Fernández

SOLUTION

SOLUTIONS. Note: only the solution to the even numbered exercises is given. The solution of the odd numbered ones is in the back of the textbook.

1. Do exercises **77 and 80** from **Section 1.2** in the book.
80: **a.** Domain: $(-\infty, \infty)$. **b.** Range: $[0, \infty)$. **c.** x-intercept: -1. **d.** y-intercept: 1. **e.** $f(-4) = 3$; $f(3) = 4$.

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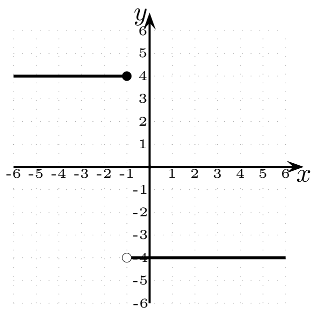
2. Do exercises **5, 6, 47 and 50** from **Section 1.3** in the book.
6: **a.** Increasing in $[-3, 2]$. **b.** Never decreasing. **c.** Never constant.
50: **a.**



b. Range: the values 4 and -4 (in set notation, $\{-4, 4\}$).

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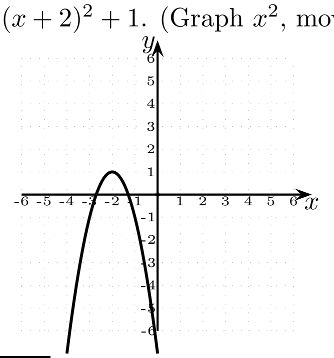
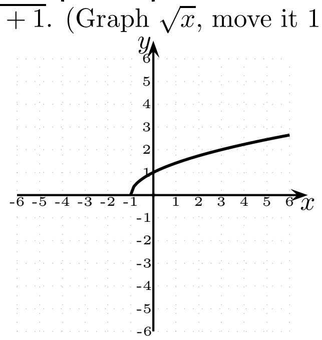
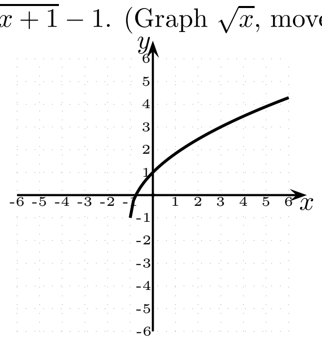
3. Do exercises **66, 70 and 80** from **Section 1.6** in the book.
66: $h(x) = -2(x+2)^2 + 1$. (Graph x^2, move it 2 left, stretch vertically by 2, reflect w.r.t. x-axis, then 1 up.)



70: $h(x) = \sqrt{x+1}$. (Graph $\sqrt{x}$, move it 1 left.)



80: $h(x) = 2\sqrt{x+1} - 1$. (Graph $\sqrt{x}$, move it 1 left, stretch vertically by 2, move it 1 down.)



4. Do exercises **67, 68 and 70** from **Section 1.7** in the book.

68. $f(x) = \frac{x}{x+5}$, $g(x) = \frac{6}{x}$.

a. $(f \circ g)(x) = f(g(x)) = \frac{\frac{6}{x}}{\frac{6}{x}+5} = \frac{6}{5x+6}$

b. To find the domain of $f \circ g$, exclude those values of x not in the domain of g and those values of x such that $g(x)$ is not in the domain of f.

Domain of g: every number excluding 0. Domain of f: every number excluding -5. For $g(x)$ not to be in the domain of f we solve $g(x) = -5$, so $\frac{6}{x} = -5$, so $x = -\frac{6}{5}$. Therefore the domain of $f \circ g$ is $(-\infty, -\frac{6}{5}) \cup (-\frac{6}{5}, 0) \cup (0, \infty)$.

70. $f(x) = \sqrt{x}$, $g(x) = x - 3$.

a. $(f \circ g)(x) = f(g(x)) = \sqrt{x-3}$

b. To find the domain of $f \circ g$, exclude those values of x not in the domain of g and those values of x such that $g(x)$ is not in the domain of f.

Domain of g: every number (nothing excluded). Domain of f: every nonnegative number, so exclude all the values for which $g(x) < 0$, so $x - 3 < 0$ so $x < 3$. Therefore the domain of $f \circ g$ is $[3, \infty)$.

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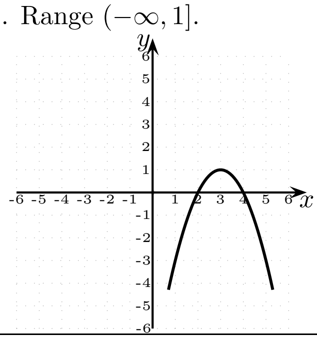
5. Do exercise **26** from **Section 2.2** in the book.

26. $f(x) = 1 - (x-3)^2$. Vertex: $(3, 1)$. Axis of symmetry: $x = 3$. x-intercepts: 4 and 2. y-intercepts: -8. Domain: $(-\infty, \infty)$. Range $(-\infty, 1]$.



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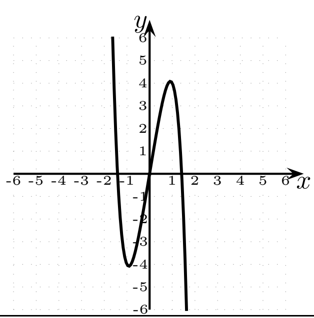
6. Do exercises **41, 47, 52** from **Section 2.3** in the book.

52. $f(x) = 6x - x^3 - x^5$. **a.** End behaviour: like $-x^5$ (up left and down right). **b.** x-intercepts: factor $f(x) = -x(x^4 + x^2 - 6) = -x(x^2 - 2)(x^2 + 3) = -x(x + \sqrt{2})(x - \sqrt{2})(x^2 + 3)$, so they are 0, $\sqrt{2}$ and $-\sqrt{2}$ all with multiplicity 1. **c.** y-intercept: $f(0) = 0$. **d.** $f(-x) = 6(-x) - (-x)^3 - (-x)^5 = -6x + x^3 + x^5 = -(6x - x^3 - x^5) = -f(x)$, so f is odd. **e.**



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7. Do exercises **11, 13, 33 and 34** from **Section 2.4** in the book.

34. $f(3) = -27$.

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8. Do exercises **2, 22, 23 and 26** from **Section 2.5** in the book.

2. Possible rational roots of $x^3 + 3x^2 - 6x - 8$ are: $\pm 1, \pm 2, \pm 4, \pm 8$.

22. $2x^3 - 5x^2 - 6x + 4 = 0$. **a.** Possible rational roots are: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}$. **b.** $\frac{1}{2}$ works, and $2x^3 - 5x^2 - 6x + 4 = 2(x - \frac{1}{2})(x^2 - 2x - 4) = 0$. So one solution is $x = \frac{1}{2}$. **c.** We find the other two by solving $(x^2 - 2x - 4) = 0$ using the quadratic formula. This gives $x = 1 + \sqrt{5}$ and $x = 1 - \sqrt{5}$.

26. If $2i$ is a zero, $-2i$ must also be a zero, and f has degree 3, so $f(x) = A(x-4)(x-2i)(x+2i)$. Now $f(-1) = 50$, so $f(-1) = A(-1-4)(-1-2i)(-1+2i) = -25A = 50$, so $A = -2$ and $f(x) = -2(x-4)(x-2i)(x+2i)$.

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9. Do exercises **51, 53, and 64** from **Section 2.6** in the book.

64. $f(x) = \frac{x-4}{x^2-x-6}$.

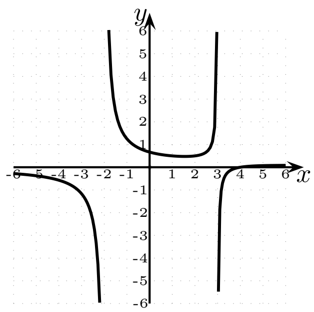
y-intercept: $f(0) = \frac{2}{3}$.

x-intercept: $x = 4$.

f cannot be even or odd since the x-intercepts are not symmetric about 0 (4 is an x-intercept but -4 is not).

Horizontal Asymptotes: $f(x) \approx \frac{x}{x^2} = \frac{1}{x}$, so f has a horizontal asymptote at $y = 0$.

Vertical Asymptotes: The denominator of $f(x)$ is $(x-3)(x+2)$, so f has vertical asymptotes at 3 and -2.



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10. Do exercises **11, 16, 53 and 56** from **Section 2.7** in the book.

16. $3x^2 + 16x < -5 \Rightarrow 3x^2 + 16x + 5 < 0 \Rightarrow (x+5)(3x+1) < 0$. Do a table of values. The solution is $(-5, -\frac{1}{3})$.

56. $\frac{x}{x-1} > 2 \Rightarrow \frac{x}{x-1} - 2 > 0 \Rightarrow \frac{-x+2}{x-1} > 0$. Do a table of values. The solution is $(1, 2)$.

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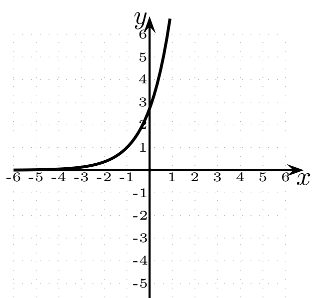
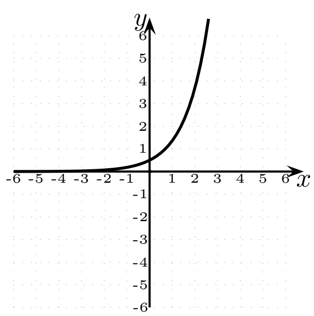
11. Do exercises **36 and 44** from **Section 3.1** in the book.

36. $f(x) = e^{x+1}$. Move the graph of e^x one unit to the left.



44. $f(x) = \frac{1}{2}e^x$. Shrink the graph of e^x vertically by 2.



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12. Do exercises **30, 32, 34, 76, 78, 86 and 90** from **Section 3.2** in the book.

30. $\log_6 \sqrt{6} = \frac{1}{2}$. **32.** $\log_3 \frac{1}{\sqrt{3}} = -\frac{1}{2}$. **34.** $\log_{81} 9 = \frac{1}{2}$.

76. Find the domain of $f(x) = \log_5(x+6)$. For something to be in the domain of $\log_b$, that something has to be positive. Therefore we need $x + 6 > 0$, so $x > -6$, so the domain of f is $(-6, \infty)$.

78. Find the domain of $f(x) = \log(7-x)$. Again, for something to be in the domain of $\log_b$, that something has to be positive. Therefore we need $7 - x > 0$, so $x < 7$, so the domain of f is $(-\infty, 7)$.

86. $10^{\log 53} = 53$. **90.** $\ln e^7 = 7$.

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13. Do exercises **40, 58, 62, 72 and 74** from **Section 3.3** in the book.

40. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right] = \log 100+3\log x+\frac{1}{3}\log(5-x)-\log 3-2\log(x+7) = 2+3\log x+\frac{1}{3}\log(5-x)-\log 3-2\log(x+7)$

58. $2\ln x - \frac{1}{2}\ln y = \ln\left(\frac{x^2}{\sqrt{y}}\right)$.

62. $4\ln x + 7\ln y - 3\ln z = \ln\left(\frac{x^4y^7}{z^3}\right)$.

72. $\log_6 17 = \frac{\log 17}{\log 6} = 1.5813$.

74. $\log_{16} 57.2 = \frac{\log 57.2}{\log 16} = 1.4595$.

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14. Do exercises **6, 8, 68 and 70** from **Section 3.4** in the book.

6. $3^{2x+1} = 27 \Rightarrow 3^{2x+1} = 3^3 \Rightarrow 2x+1 = 3 \Rightarrow x = 1$. The solution is $x = 1$.

8. $5^{3x-1} = 125 \Rightarrow 5^{3x-1} = 5^3 \Rightarrow 3x-1 = 3 \Rightarrow x = \frac{4}{3}$. The solution is $x = \frac{4}{3}$.

68. $\log_2(x-1)+\log_2(x+1) = 3 \Rightarrow \log_2(x-1)(x+1) = 3 \Rightarrow (x-1)(x+1) = 8 \Rightarrow x^2-1 = 8 \Rightarrow x^2 = 9 \Rightarrow x = \pm 3$. However, $x = -3$ does not work because $\log_2(-3-1) = \log_2(-4)$ is not defined. So the only solution is $x = 3$.

70. $\log_4(x+2)-\log_4(x-1) = 1 \Rightarrow \log_4\frac{x+2}{x-1} = 1 \Rightarrow \frac{x+2}{x-1} = 4 \Rightarrow \frac{x+2}{x-1} - 4 = 0 \Rightarrow \frac{-3x+6}{x-1} = 0 \Rightarrow \frac{-3(x-2)}{x-1} = 0$, so $x = 2$. Check: $\log_4(2+2)-\log_4(2-1) = \log_4 4 - \log_4 1 = 1-0 = 1$.

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15. Do exercises **25, 28, 40, 48** from **Section 4.2** in the book.

28. If $\sin t = \frac{2}{3}$ and $\cos t = \frac{\sqrt{5}}{3}$ then $\tan t = \frac{\sin t}{\cos t} = \frac{2}{\sqrt{5}}$, $\sec t = \frac{1}{\cos t} = \frac{3}{\sqrt{5}}$, $\csc t = \frac{1}{\sin t} = \frac{3}{2}$, $\cot t = \frac{\cos t}{\sin t} = \frac{\sqrt{5}}{2}$

40. $\csc\frac{9\pi}{4} = \frac{1}{\sin\frac{9\pi}{4}} = \frac{1}{\sin\frac{\pi}{4}} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$.

48. $-\cot\left(\frac{\pi}{4}+17\pi\right) = -\cot\left(\frac{\pi}{4}+\pi\right) = -\cot\left(\frac{5\pi}{4}\right) = -\frac{\cos\left(\frac{5\pi}{4}\right)}{\sin\left(\frac{5\pi}{4}\right)} = -\frac{-\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = -1$ (the first equality comes from the fact that $\left(\frac{\pi}{4}+17\pi\right)$ and $\left(\frac{\pi}{4}+\pi\right)$ are coterminal angles).

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16. Do exercises **10, 12, 14, 44 and 54** from **Section 4.3** in the book.

10. $\tan 30° = \frac{1}{\sqrt{3}}$.

12. $\csc 45° = \frac{\sqrt{2}}{1} = \sqrt{2}$.

14. $\cot\frac{\pi}{3} = \cot 60° = \frac{1}{\sqrt{3}}$.

44. $\frac{1}{\cot\frac{\pi}{4}} - \frac{2}{\csc\frac{\pi}{6}} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} - 2\sin\frac{\pi}{6} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} - 2\cdot\frac{1}{2} = 1-1 = 0$.

54. By looking at the picture, $\tan 40° = \frac{h}{35}$, so $h = 35\tan 40° \approx 29ft$.

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17. Do exercises **25, 26, 68, 72 and 74** from **Section 4.4** in the book.

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26. Suppose $\cos\theta = \frac{4}{5}$. Draw a right triangle whose leg adjacent to θ has length 4 and whose hypotenuse has length 5. Then the leg opposite to θ has lenght $\sqrt{5^2-4^2} = \sqrt{25-16} = \sqrt{9} = 3$ by the Pythagorean theorem. So $\sin\theta = \pm\frac{3}{5}$. Since θ is in quadrant IV, $\sin\theta$ is negative, so $\sin\theta = -\frac{3}{5}$ and from here $\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{3}{4}$, $\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$, $\csc\theta = \frac{1}{\sin\theta} = -\frac{5}{3}$, and $\cot\theta = \frac{\cos\theta}{\sin\theta} = -\frac{4}{3}$.

68. $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.

72. $\tan\frac{9\pi}{2} = \tan\frac{8\pi+\pi}{2} = \tan\left(\frac{8\pi}{2}+\frac{\pi}{2}\right) = \tan\left(4\pi+\frac{\pi}{2}\right) = \tan\frac{\pi}{2}$, which is undefined.

74. $\sin(-225°) = \sin(-225°+360°) = \sin(135°) = \sqrt{2}2$.

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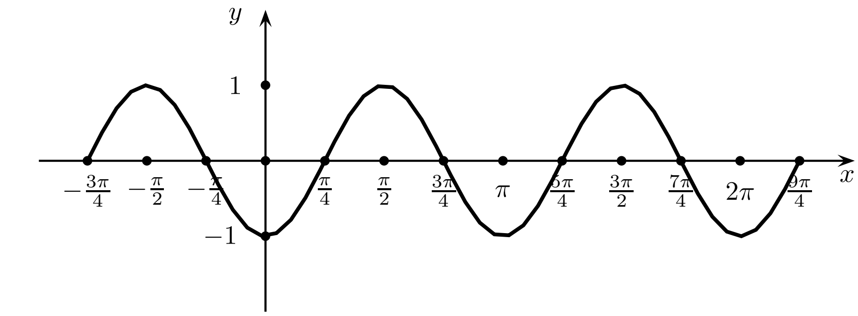
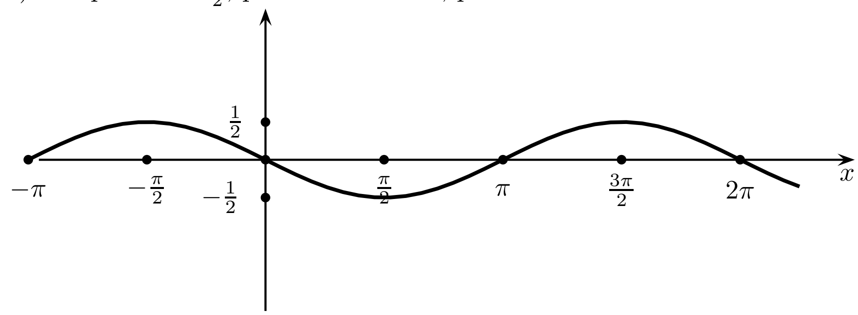
18. Do exercises **17, 20, 24 and 26** from **Section 4.5** in the book.

20. $y = \sin(2x-\frac{\pi}{2})$. Amplitude = 1, phase shift = $\frac{\pi}{4}$, period = π.



24. $y = \frac{1}{2}\sin(x+\pi)$. Amplitude $= \frac{1}{2}$, phase shift $= -\pi$, period $= 2\pi$.



26. $y = -3\sin\left(2x+\frac{\pi}{2}\right)$. Amplitude $= 3$, phase shift $= -\frac{\pi}{4}$, period $= \pi$.