Exercises.

(1) Which of the following functions are one-to-one?
  (a) $f(x) = 42$
  (b) $g(x) = -2x + 5$
  (c) $h(x) = x^2 - 3$
  (d) $f(x) = x^2 + 1$ with domain $[0, \infty)$.
  (e) $g(x) = x^3$
  (f) $h(x) = x^3 - 8$
  (g) $g(x) = \sqrt{x+1}$
  (h) $f(x) = \sqrt{1 - x^2}$
  (i) $f(x) = (x-2)^3$
  (j) $h(x) = \dfrac{3}{2x-4}$
  (k) $g(x) = \dfrac{3x+6}{x+1}$
  (l) $f(x) = \dfrac{2x-3}{5x-2}$
  (m) $f(x) = x^2 - 3x + 2$
  (n) $g(x) = x^2 + 2x + 4$
  (o) $h(x) = \sin x$
  (p) $f(x) = 2^{x+1}$
  (q) $g(x) = \log_2(x-1)$
  (r) Extra Credit $f(x) = (x-1)(x-2)(x-3)$.

(2) For each of the functions of the above exercise
  - if the function is one-to-one find the inverse function.
  - if the function is not one-to-one then find a maximal interval so that restricting the function to that interval makes it one-to-one.

(3) Give an example of a relation that is not a function but its inverse is a function.

(4) Prove that if a function is one-to-one then its inverse function is also one-to-one.

(5) A function is called *even*, if the following two conditions are satisfied:
  - The domain of the function is "symmetric around 0", that is if a number a is in the domain then $-a$ is also in the domain.
  - For all a in the domain, $f(a) = f(-a)$.

  Can you give an example of an even function that is one-to-one?

(6) A function is called *odd*, if the following two conditions are satisfied:
  - The domain of the function is "symmetric around 0", that is if a number a is in the domain then $-a$ is also in the domain.
  - For all a in the domain, $f(a) = -f(-a)$.

  Can you give an example of an odd function that is one-to-one?