MATH 30 - Precalculus. Homework 4. Due Th. 02/26/2025. Professor Luis Fernández

SOLUTION

DO NOT write your answers here. Do it in other sheets and **show all your work**. **STAPLE this sheet to your other sheets.**

- 1. Use synthetic division and the remainder theorem to find the indicated function value.
 - a) $f(x) = x^3 4x^2 + x + 2$; find f(3). b) $f(x) = -2x^4 - x^2 + x - 2$; find f(-1). c) $f(x) = x^5 - 4x^2 + 1$; find f(2). d) $f(x) = -x^4 - 5x^3 - x^2 + 3x + 2$; find $f\left(\frac{1}{2}\right)$.

Solution:

Synthetic divisions skipped. For example, you can use http://www.mathcelebrity.com/syndiv.php to check it.

a) Answer: f(3) = -4.
b) Answer: f(-1) = -6.
c) Answer: f(2) = 17.
d) Answer: f(1/2) = 41/16.

2. Find the possible rational zeros of the following polynomials.

a) $4x^3 + 5x^2 - 3x + 6$ Solution: a) $\pm \{1, 2, 3, 6, \frac{1}{2}, \frac{1}{4}, \frac{3}{2}, \frac{3}{4}\}$ b) $\pm \{1, 3, 5, 15, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{3}{2}, \frac{5}{2}, \frac{5}{3}, \frac{5}{6}, \frac{15}{2}\}$

- 3. Solve the following polynomial equations. (We did several examples in class.)
 - a) $x^3 4x^2 7x + 10 = 0$ b) $3x^3 - 8x^2 - 8x + 8 = 0$ c) $x^4 + 3x^3 - 20x^2 + 24x - 8 = 0$ d) $x^4 - x^3 + 2x^2 - 4x - 8 = 0$ Solution:
- a) The possible rational solutions of $x^3 4x^2 7x + 10 = 0$ are $\pm 1, \pm 2, \pm 5, \pm 10$. Now we do synthetic division to test them. Check that 1 is not a root. However, -1 is:

	1 - 4 - 7 10	
1	1 -3 -10	
	1 -3 -10 0	
-2	-2 10	
	1 -5 0	
5	5	Therefore the solutions are $1, -2$ and 5 .
	1 0	

b) I only write the solutions (proceed as in the previous exercise, or as in exercise 2). They are $\frac{2}{3}$, $1 + \sqrt{5}$, $1 - \sqrt{5}$.

c) I only write the solutions (proceed as in the previous exercise, or as in exercise 2). They are: 1, 2, $-3 - \sqrt{13}$, $-3 + \sqrt{13}$.

- d) I only write the solutions (proceed as in the previous exercise, or as in exercise 2). They are -1, 2, 2i, -2i.
- 4. Use the results of the previous exercise to factor the following polynomials completely. [NOTE: you DO NOT need to do any calculation, only use the *factor theorem*.]

a) $x^3 - 4x^2 - 7x + 10$ b) $3x^3 - 8x^2 - 8x + 8$ c) $x^4 + 3x^3 - 20x^2 + 24x - 8$

d)
$$x^4 - x^3 + 2x^2 - 4x - 8$$

Solution:

- a) From the previous exercise, the zeros of $x^3 4x^2 7x + 10$ are 1, -2 and 5. Therefore, $x^3 - 4x^2 - 7x + 10 = (x - 1)(x + 2)(x - 5)$
- **b)** From the previous exercise, the zeros of $3x^3 8x^2 8x + 8$ are $\frac{2}{3}$, $1 + \sqrt{5}$, $1 \sqrt{5}$. Therefore, $3x^3 8x^2 8x + 8 = (x \frac{2}{3})(x (1 + \sqrt{5}))(x (1 \sqrt{5}))$.
- c) From the previous exercise, the zeros of $x^4 + 3x^3 20x^2 + 24x 8$ are 1, 2, $-3 \sqrt{13}$, $-3 + \sqrt{13}$. Therefore, $x^4 + 3x^3 - 20x^2 + 24x - 8 = (x - 1)(x - 2)(x - (-3 - \sqrt{13}))(x - (-3 + \sqrt{13}))$
- d) From the previous exercise, the zeros of $x^4 x^3 + 2x^2 4x 8$ are -1, 2, 2i, -2i. Therefore, $x^4 - x^3 + 2x^2 - 4x - 8 = (x+1)(x-2)(x+2i)(x-2i)$.
- 5. Solve the equation $(x-1)^2(x-2)(x-3)(x+4) = 0$. [NOTE: you DO NOT need to do any calculation for this one; use the factor theorem to find the solution by just looking at the equation.] Solution: 1 (with multiplicity two), 2, 3 and -4.
- 6. For the following polynomials, find the end behavior, the y intercept, and the x intercepts with multiplicities. Then sketch the graph in graph paper.
 - **b)** $f(x) = x^3 x^2 5x 3$ a) $f(x) = (x-2)^2(x+1)^3(x-1)$