MATH 30 - Precalculus. Homework 3. Due Th. 02/20/2025. Professor Luis Fernández

SOLUTION

DO NOT write your answers here. Do it in other sheets and **show all your work**.

STAPLE this sheet to your other sheets.

1. Divide using long division. State the quotient $q(x)$ and the remainder r. Then write the solution in two different ways:

1. As $D = dq + r$.
2. As $\dfrac{D}{d} = q + \dfrac{r}{d}$.

[Where D is the dividend (the polynomial that is being divided; in other words, the numerator) and d is the divisor (the polynomial that divides; in other words, the denominator).]

a) $\dfrac{x^3 - 2x^2 - 5x + 6}{x + 2}$

b) $\dfrac{3x^4 - 2x^3 - 7x^2 + x - 2}{x^2 - 2x + 3}$

c) $\dfrac{-2x^3 - 7x^2 + x - 2}{x^2 - x + 2}$

d) $\dfrac{x^7 - 1}{x - 1}$

Solution:

a)

$$
\begin{array}{r|l}
 & x^2 - 4x + 3 \\
\hline
x + 2 & x^3 - 2x^2 - 5x + 6 \\
- & x^3 + 2x^2 \\
\hline
 & -4x^2 - 5x \\
- & -4x^2 - 8x \\
\hline
 & 3x + 6 \\
- & 3x + 6 \\
\hline
 & 0
\end{array}
$$

Therefore $q = x^2 - 4x + 3$ and $r = 0$

1. $x^3 - 2x^2 - 5x + 6 = (x + 2)(x^2 - 4x + 3) + 0$
2. $\dfrac{x^3 - 2x^2 - 5x + 6}{x^2 - 4x + 3} = (x + 2)$

b)

$$
\begin{array}{r|l}
 & 3x^2 + 4x - 8 \\
\hline
x^2 - 2x + 3 & 3x^4 - 2x^3 - 7x^2 + x - 2 \\
- & 3x^4 - 6x^3 + 9x^2 \\
\hline
 & 4x^3 - 16x^2 + x \\
- & 4x^3 - 8x^2 + 12x \\
\hline
 & -8x^2 - 11x - 2 \\
- & -8x^2 + 16x - 24 \\
\hline
 & -27x + 22
\end{array}
$$

Therefore $q = 3x^2 + 4x - 8$ and $r = -27x + 22$

1. $3x^4 - 2x^3 - 7x^2 + x - 2$
   $= (x^2 - 2x + 3)(3x^2 + 4x - 8) + (-27x + 22)$
2. $\dfrac{3x^4 - 2x^3 - 7x^2 + x - 2}{x^2 - 2x + 3}$
   $= (3x^2 + 4x - 8) + \dfrac{-27x + 22}{x^2 - 2x + 3}$

c) (Division skipped. But you can check your answer, for example, at https://www.emathhelp.net/en/calculators/algebra-1/polynomial-long-division-calculator/).

Answer: $q = -2x - 9$ and $r = -4x + 16$, so

1. $-2x^3 - 7x^2 + x - 2 = (x^2 - x + 2)(-2x - 9) + (-4x + 16)$
2. $\dfrac{-2x^3 - 7x^2 + x - 2}{x^2 - x + 2} = (-2x - 9) + \dfrac{-4x + 16}{x^2 - x + 2}$

d)

$$
\begin{array}{rrrrrrrrrrrrrrrrr}
 & x^7 & + & x^6 & + & x^5 & + & x^4 & + & x^3 & + & x^2 & + & x & + & 1 \\
\hline
x-1 \;\Big| & x^7 & + & 0\cdot x^6 & + & 0\cdot x^5 & + & 0\cdot x^4 & + & 0\cdot x^3 & + & 0\cdot x^2 & + & 0\cdot x^1 & - & 1 \\
- & x^7 & - & x^6 & & & & & & & & & & & & \\
\hline
 & & - & x^6 & + & 0\cdot x^5 & & & & & & & & & & \\
 & & & x^6 & - & x^5 & & & & & & & & & & \\
\hline
 & & & & - & x^5 & + & 0\cdot x^4 & & & & & & & & \\
 & & & & & x^5 & - & x^4 & & & & & & & & \\
\hline
 & & & & & & - & x^4 & + & 0\cdot x^3 & & & & & & \\
 & & & & & & & x^4 & - & x^3 & & & & & & \\
\hline
 & & & & & & & & - & x^3 & + & 0\cdot x^2 & + & & & \\
 & & & & & & & & & x^3 & - & x^2 & & & & \\
\hline
 & & & & & & & & & & - & x^2 & + & 0\cdot x & & \\
 & & & & & & & & & & & x^2 & - & x & & \\
\hline
 & & & & & & & & & & & & - & x & - & 1 \\
 & & & & & & & & & & & & & x & - & 1 \\
\hline
 & & & & & & & & & & & & & & & 0
\end{array}
$$

Answer: $q = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ and $r = 0$, so

1. $x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
2. $\dfrac{x^7-1}{x-1} = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

---

2. Divide using synthetic division. State the quotient q and the remainder r. Then write the solution in two different ways:

1. As $D = dq + r$.
2. As $\dfrac{D}{d} = q + \dfrac{r}{d}$.

[Where D is the dividend (the polynomial that is being divided; in other words, the numerator) and d is the divisor (the polynomial that divides; in other words, the denominator).]

a) $\dfrac{x^3 - 2x^2 - 5x + 6}{x-3}$

b) $\dfrac{-2x^3 - 7x^2 + x - 2}{x+1}$

c) $\dfrac{x^4 - x^3 + x - 1}{x-2}$

d) $\dfrac{x^7-1}{x-1}$

Solution:

a)

$$
\begin{array}{r|rrrr}
 & 1 & -2 & 5 & 6 \\
3 & & 3 & 3 & 24 \\
\hline
 & 1 & 1 & 8 & 30
\end{array}
$$

So $q = x^2 + x + 8$ and $r = 30$.

1. $x^3 - 2x^2 - 5x + 6 = (x-3)(x^2 + x + 8) + 30$
2. $\dfrac{x^3 - 2x^2 - 5x + 6}{x-3} = x^2 + x + 8 + \dfrac{30}{x-3}$

b)

$$
\begin{array}{r|rrrr}
 & -2 & -7 & 1 & -2 \\
-1 & & 2 & 5 & -6 \\
\hline
 & -2 & -5 & 6 & -8
\end{array}
$$

So $q = -2x^2 - 5x + 6$ and $r = -8$.

1. $-2x^3 - 7x^2 + x - 2 = (x+1)(-2x^2 - 5x + 6) - 8$
2. $\dfrac{-2x^3 - 7x^2 + x - 2}{x+1} = -2x^2 - 5x + 6 + \dfrac{-8}{x+1}$

c)

$$
\begin{array}{r|rrrrr}
 & 1 & -1 & 0 & 1 & -1 \\
2 & & 2 & 2 & 4 & 10 \\
\hline
 & 1 & 1 & 2 & 5 & 9
\end{array}
$$

So $q = x^3 + x^2 + 2x + 5$ and $r = 9$.

1. $x^4 - x^3 + x - 1 = (x-2)(x^3 + x^2 + 2x + 5) + 9$
2. $\dfrac{x^4 - x^3 + x - 1}{x-2} = x^3 + x^2 + 2x + 5 + \dfrac{9}{x-2}$

d)

$$
\begin{array}{r|rrrrrrrr}
 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\hline
 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0
\end{array}
$$

So $q = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$ and $r = 0$.

1. $x^7 - 1 = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1)$
2. $\dfrac{x^7-1}{x-1} = x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$