SOLUTION

**1.** For the following quadratic functions,
- Find the vertex and x- and y-intercepts.
- Give the equation of the axes of symmetry.
- Draw the graph in the axes provided,
- Determine the function's domain and range.

a) $f(x) = (x-4)^2 - 1$.
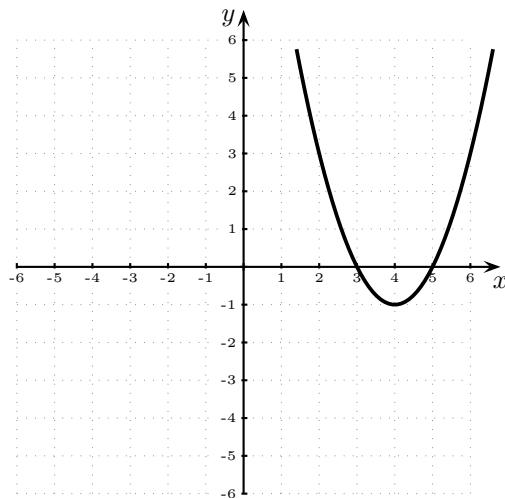
Vertex: $(4, -1)$
y-**intercept:** $15.$ x-**intercepts:** $3, 5$
Axes of symmetry: $x = 4$
Domain: $\mathbb{R}$.   **Range:** $[-1, \infty)$

b) $g(x) = 4 - (x-1)^2$.
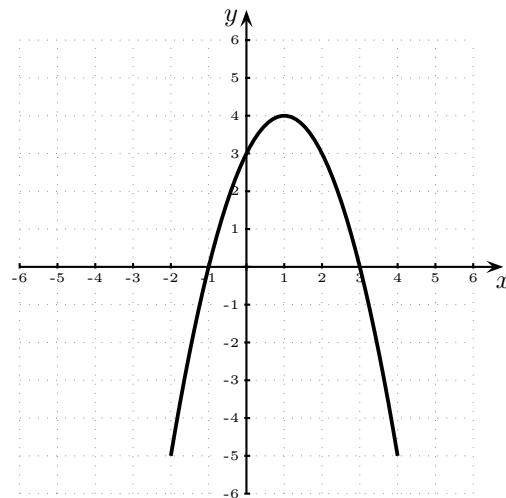
Vertex: $(1, 4)$
y-**intercept:** $3.$ x-**intercepts:** $-1, 3$
Axes of symmetry: $x = 1$
Domain: $\mathbb{R}$.   **Range:** $(-\infty, 4]$

c) $h(x) = 3x^2 - 2x - 4$.
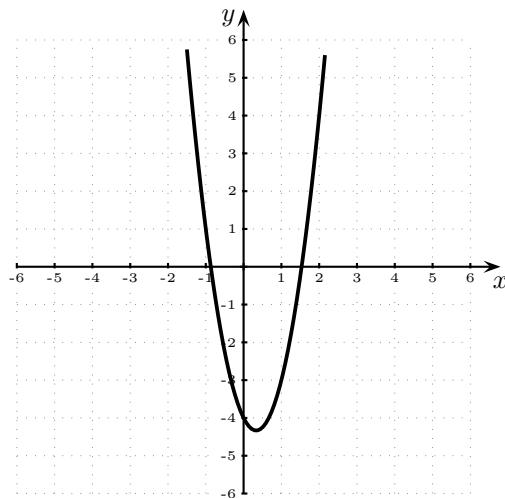
Vertex: $(1/3, -13/3)$
y-**intercept:** $-4.$ x-**intercepts:** $\dfrac{1 \pm \sqrt{13}}{3}$
Axes of symmetry: $x = 1/3$
Domain: $\mathbb{R}$.   **Range:** $[-13/3, \infty)$

d) $i(x) = 2x - x^2 - 2$.
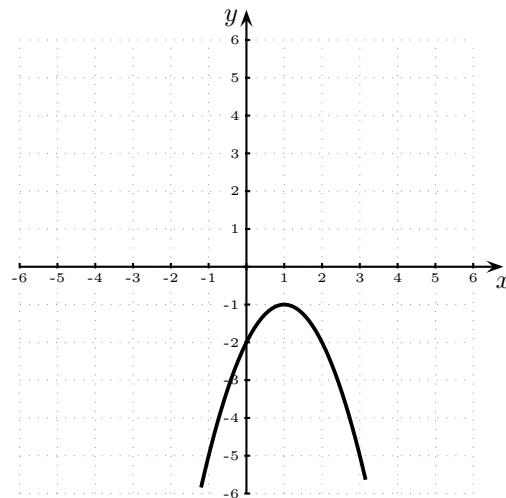
Vertex: $(1, -1)$
y-**intercept:** $-2.$ x-**intercepts:** NONE
Axes of symmetry: $x = 1$
Domain: $\mathbb{R}$.   **Range:** $(-\infty, 1]$

2. For each of the following functions, find

(i) The end behaviour of the graph.

(ii) The y-intercept.

(iii) For exercises **a)**, **b)**, **c)**, the x-intercepts with their multiplicity and the local behaviour at the x-intercepts.

(iv) Do the graphs of all the functions using any graphing device. For example, use https://www.desmos.com/calculator
Check that the end behaviour of the graphs that you found in part **(i)** are all correct.

a) $f(x) = 2(x-2)^2(x+1)$ 　　　　　　　　　　**b)** $f(x) = -2x^2(x-2)(x+2)^2$

c) $f(x) = 3x(x+1)^2(x-1)^3$ 　　　　　　　　　**d)** $f(x) = -x^4 + 5x^2 + x$

Solution:

(i) End behavior: **a)** ↙ ↗ **b)** ↖ ↘ **c)** ↖ ↗ **d)** ↙ ↘

(ii) y intercept: **a)** $y = 8$. **b)** $y = 0$. **c)** $y = 0$. **d)** $y = 0$.

(iii) x intercepts:

　　a) $x = 2$, multiplicity 2, ⌣ or ⌢; $x = -1$, multiplicity 1, ↗ or ↘ .

　　b) $x = 0$, multiplicity 2, ⌣ or ⌢; $x = 2$; mult. 1, ↗ or ↘ ; $x = -2$; mult. 2, ⌣ or ⌢.

　　c) $x = 0$, multiplicity 1, ↗ or ↘ ; $x = -1$, multiplicity 2, ⌣ or ⌢; $x = 1$, mult. 3, ↗ or ↘.

Do the graphs with your favorite graphing device to see the solution.