MATH 30 - Precalculus. Homework 12. Not to hand in. Professor Luis Fernández

SOLUTION

DO NOT write your answers here. Do it in other sheets and **show all your work**.

STAPLE this sheet to your other sheets.

1. Recall that to show that a function g is the inverse of a function f one needs to show that $f(g(x)) = x$ and that $g(f(x)) = x$. To do this,

1. Find $f(g(x))$ and simplify and see that you get x.
2. Find $g(f(x))$ and simplify and see that you get x.

For the following, show that g is the inverse of f.

a) $f(x) = 4x - 7$ and $g(x) = \dfrac{x+7}{4}$.

b) $f(x) = \dfrac{2}{x-5}$ and $g(x) = \dfrac{2}{x} + 5$.

c) $f(x) = -3x + 1$ and $g(x) = \dfrac{x-1}{-3}$.

d) $f(x) = \dfrac{x-2}{2x+1}$ and $g(x) = \dfrac{-x-2}{2x-1} + 5$.

Solution:

a) 1. $f(g(x)) = f(\frac{x+7}{4}) = 4 \cdot \frac{x+7}{4} - 7 = (x+7) - 7 = x$. YES.

2. $g(f(x)) = g(4x-7) = \dfrac{(4x-7)+7}{4} = \frac{4x}{4} = x$. YES.

b) 1. $f(g(x)) = f(\frac{2}{x} + 5) = \dfrac{2}{\left(\dfrac{2}{x} + 5\right) - 5} = \dfrac{2}{\dfrac{2}{x}} = x$. YES.

2. $g(f(x)) = g(\frac{2}{x-5}) = \dfrac{2}{\dfrac{2}{x-5}} + 5 = \dfrac{2(x-5)}{2} + 5 = (x-5) + 5 = x$. YES.

c) and **d)**: proceed in the same way.

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2. Find the inverse of the following functions.

a) $f(x) = 2x - 1$

b) $g(x) = \dfrac{1}{x} + 1$

c) $h(x) = x^2 - 4$, with domain $(-\infty, 0]$ (so $x \leq 0$)

d) $i(x) = \dfrac{x-1}{x+1}$.

Solution:

a) We need to solve $f(y) = x$ for y, i.e. solve $2y - 1 = x$ for y.

$$2y - 1 = x \quad \Rightarrow \quad 2y = x + 1 \quad \Rightarrow \quad y = \frac{x+1}{2}.$$

Therefore the inverse of f is $f^{-1}(x) = \frac{x+1}{2}$ (or, if you prefer, $f^{-1}(y) = \frac{y+1}{2}$).

b) We need to solve $g(y) = x$ for y, i.e. solve $\frac{1}{y} + 1 = x$ for y.

$$\frac{1}{y} + 1 = x \quad \Rightarrow \quad \frac{1}{y} = x - 1 \quad \Rightarrow \quad y = \frac{1}{x-1}.$$

Therefore the inverse of g is $g^{-1}(x) = \frac{1}{x-1}$.

c) We need to solve $h(y) = x$ for y, i.e. solve $y^2 - 4 = x$ for y, where we know that $y \leq 0$.

$$y^2 - 4 = x \quad \Rightarrow \quad y^2 = x + 4 \quad \Rightarrow \quad y = \pm\sqrt{x+4},$$

but since we know that y is negative, the only possible solution is $y = -\sqrt{x+4}$. Therefore the inverse of h is $h^{-1}(x) = -\sqrt{x+4}$.

d) We need to solve $i(y) = x$ for y, i.e. solve $\frac{y-1}{y+1} = x$ for y.

$$\frac{y-1}{y+1} = x \quad \Rightarrow \quad y - 1 = x(y+1) \quad \Rightarrow \quad y - 1 = xy + x \quad \Rightarrow \quad y - xy = x + 1 \quad \Rightarrow \quad y(1-x) = x + 1 \quad \Rightarrow \quad y = \frac{x+1}{1-x}.$$

Therefore the inverse of i is $i^{-1}(x) = \frac{x+1}{1-x}$.

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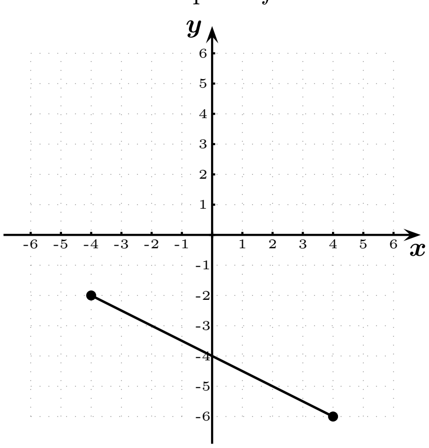
3. Let f be the function described by the following graph:

Graph of f



a) Fill in the blanks (using interval notation):

The domain of f is $[-4, 4]$ The range of f is $[-6, -2]$

The domain of f^{-1} is $[-6, -2]$ The range of f^{-1} is $[-4, 4]$

We can see that the domain of f is the same as the **range** of f^{-1},
and the range of f is the same as the domain of f^{-1}

b) Evaluate the following:

$f^{-1}(-3) = -2$ $f^{-1}(-4) =0$ $f^{-1}(-6) = 2$

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4. Solve the following equations.

a) $|x - 3| = 4$. **Solution:** $x - 3 = 4$ or $x - 3 = -4$, which gives $x = 7$ or $x = -1$.
b) $|x + 2| = 5$. **Solution:** $x + 2 = 5$ or $x + 2 = -5$, which gives $x = 3$ or $x = -7$.
c) $|2x + 3| = 9$. **Solution:** $2x + 3 = 9$ or $2x + 3 = -9$, which gives $x = 3$ or $x = -6$.

5. Solve the following inequalities.

a) $|x-3| \le 4$. **Solution:** $-4 \le x-3 \le 4$, which gives $-1 \le x \le 7$, so the solution is $[-1, 7]$.

b) $|x+2| \ge 5$. **Solution:** $x+2 \ge 5$ or $x+2 \le -5$, which gives $x \ge 3$ or $x \le -7$, so the solution is $(-\infty, -7] \cup [3, \infty)$.

c) $|2x+3| > 9$. **Solution:** $2x+3 > 9$ or $2x+3 < -9$, which gives $x > 3$ or $x < -6$, so the solution is $(-\infty, -6) \cup (3, \infty)$.

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6. Find the following values of inverse trigonometric functions.

a) $\sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$ **b)** $\sin^{-1}(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$ **c)** $\sin^{-1}(-\frac{\sqrt{1}}{2}) = -\frac{\pi}{6}$

d) $\sin^{-1}(-1) = -\frac{\pi}{2}$ **e)** $\sin^{-1}(1) = \frac{\pi}{2}$ **f)** $\cos^{-1}(-\frac{\sqrt{1}}{2}) = \frac{2\pi}{3}$