

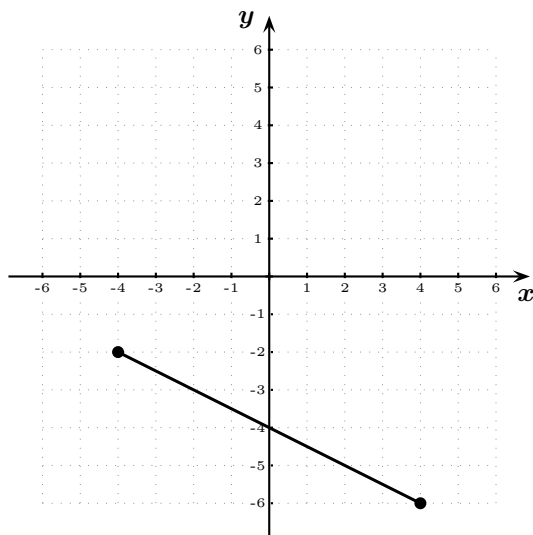
SOLUTION

DO NOT write your answers here. Do it in other sheets and **show all your work**.

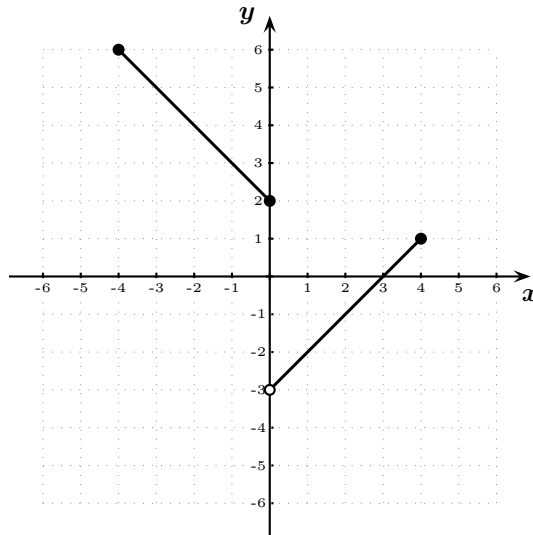
STAPLE this sheet to your other sheets.

1. Let f and g be the functions described by the following graphs:

Graph of f



Graph of g



- a) Fill in the blanks (using interval notation):

The domain of f is $[-4, 4]$

The range of f is $[-6, -2]$

The domain of g is $[-4, 4]$

The range of g is $(-3, 1] \cup [2, 6]$

An interval on which g is one-to-one is: $[-4, 4]$ (g is injective!)

- b) Evaluate the following, if they exist:

$$g(0) = 2$$

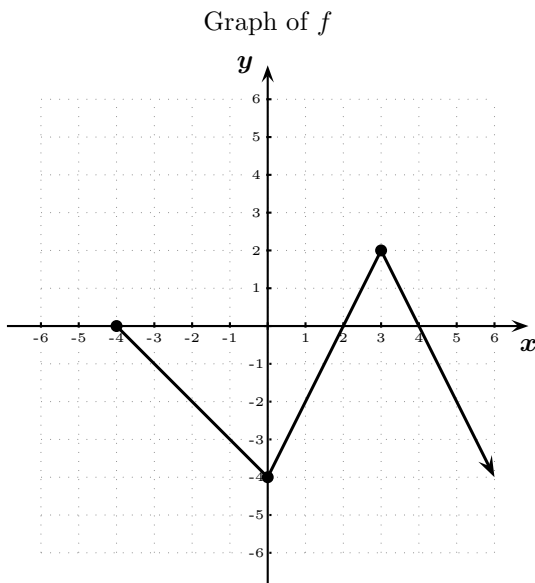
$$(f + g)(-2) = f(-2) + g(-2) = -3 + 4 = 1$$

$$\left(\frac{g}{f}\right)(-2) = \frac{4}{-3}$$

$$(g \circ f)(-4) = g(f(-4)) = g(-2) = 4$$

$$(f \circ f)(0) = f(f(0)) = f(-4) = -2$$

2. Let f be the function described by the following graph:



a) Fill in the blanks (using interval notation):

The domain of f is $(-4, \infty)$

The range of f is $(-\infty, 2]$

Write the set of all x where f is negative in interval notation: $(-4, 2) \cup (4, \infty)$

Write the largest open interval of x where f is increasing: $(0, 3)$

b) Is f a one-to-one function? (Answer yes or no): No

c) The x -intercepts of f are -4 , 2 and 4 . The y -intercept of f is -4

d) f has a local minimum at $x = 0$

e) Evaluate the following, if they exist:

$$f(-1) = -3$$

$$f(3) = 2$$

3. Given $f(x) = 2x + 1$ and $g(x) = x^2 + 3$, find and simplify:

a) $(f \circ g)(3)$

b) $(f \circ g)(x)$

c) $(g \circ f)(x)$

d) $(f \circ g \circ f)(x)$

Solution:

a) $(f \circ g)(3) = f(g(3)) = f(12) = 25$

b) $(f \circ g)(x) = f(g(x)) = f(x^2 + 3) = 2(x^2 + 3) + 1 = 2x^2 + 7$

c) $(g \circ f)(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 + 3 = 4x^2 + 4x + 4 = 4(x^2 + x + 1)$

d) $(f \circ g \circ f)(x) = f(g(f(x))) = f(g(2x + 1)) = f((2x + 1)^2 + 3) = 2((2x + 1)^2 + 3) + 1 = 8x^2 + 8x + 9$

4. Given $f(x) = \frac{1}{x}$ and $g(x) = \frac{2}{x-1}$, find and simplify

a) $(f \circ g)(3)$

b) $(f \circ g)(x)$

c) $(g \circ f)(x)$

d) $(f \circ g \circ f)(x)$

Solution:

a) $(f \circ g)(3) = f(g(3)) = f(1) = 1$

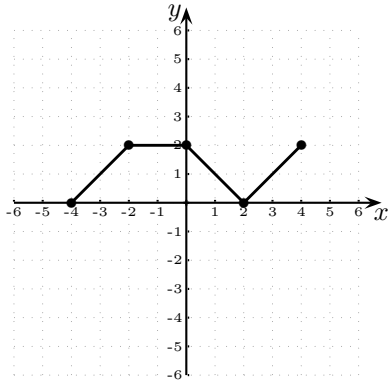
b) $(f \circ g)(x) = f(g(x)) = f\left(\frac{2}{x-1}\right) = \frac{1}{\frac{2}{x-1}} = \frac{x-1}{2}$.

c) $(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{2}{\frac{1}{x}-1} = \frac{2x}{1-x}$

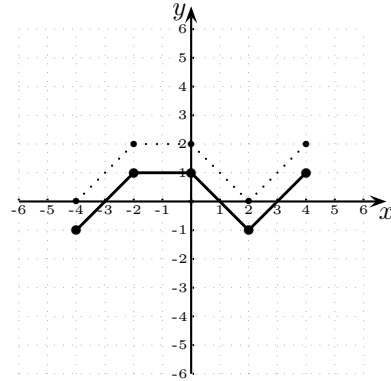
d) $(f \circ g \circ f)(x) = f(g(f(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{2}{\frac{1}{x}-1}\right) = f\left(\frac{2x}{1-x}\right) = \frac{1}{\frac{2x}{1-x}} = \frac{1-x}{2x}$

5. Use the graph of $y = f(x)$ to graph each function g . You can use the axes provided in this sheet.

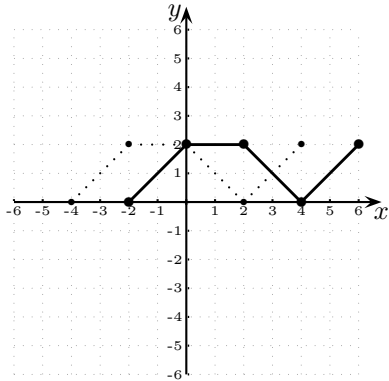
This is the given original graph of f .



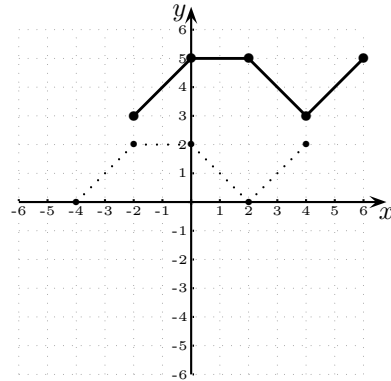
a) Graph $g(x) = f(x) - 1$.



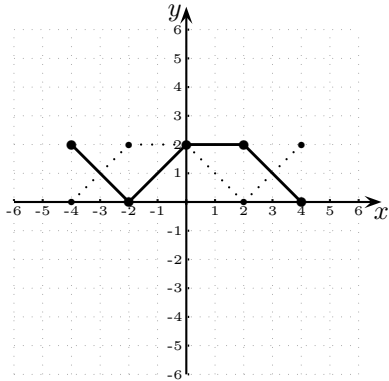
b) Graph $g(x) = f(x - 2)$.



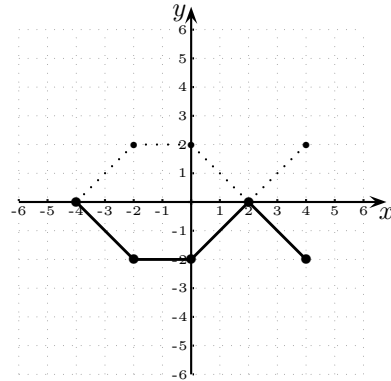
c) Graph $g(x) = f(x - 2) + 3$.



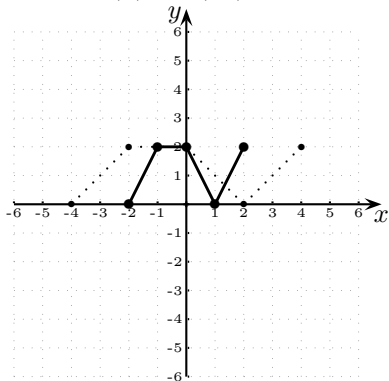
d) Graph $g(x) = f(-x)$.



e) Graph $g(x) = -f(x)$.



f) Graph $g(x) = f(2x)$.



g) Graph $g(x) = 2f(x)$.

