MATH 30 - Precalculus. Homework 6. Due We. 03/20/2024. Professor Luis Fernández

SOLUTION

DO NOT write your answers here, except the graphs. Do it in other sheets and **show all your work**.
STAPLE this sheet to your other sheets.

1. Use the properties of logarithms to expand the following expressions.

a) $\log_9(5y) = \log_9 5 + \log_9 y$

b) $\log_8 x^7 = 7\log_8 x$

c) $\log_b(3x^2y^3) = \log_b 3 + 2\log_b x + 3\log_b y$

d) $\log_8 \dfrac{x^{\frac{1}{2}}}{y^3} = \dfrac{1}{2}\log_8 x - 3\log_8 y$

e) $\log_5 \sqrt[5]{\dfrac{x^2}{y}} = \dfrac{1}{5}(2\log_5 x - \log y)$

f) **g)** $\ln\left[\dfrac{x^4\sqrt{x^2+3}}{(x+3)^5}\right] = 4\ln x + \frac{1}{2}\ln(x^2+3) - 5\ln(x+3)$

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2. Use the properties of logarithms to condense the following expressions.

a) $\log x + \log 5 = \log 5x$

b) $\log_8 x + 3\log_8 y = \log_8(xy^3)$

c) $4\,\mathrm{Ln}(x+6) - 5\,\mathrm{Ln}(x+1) = \mathrm{Ln}\left(\dfrac{(x+6)^4}{(x+1)^5}\right)$

d) $2\log x + 3\log y - 4\log z = \log\left(\dfrac{x^2y^3}{z^4}\right)$

e) $\frac{1}{2}(\log x + \log y) = \log\sqrt{xy}$

f) $\frac{1}{3}(\log_7 x + 4\log_7 y) - 3\log_7(x+y) = \log_7 \dfrac{\sqrt[3]{xy^4}}{(x+y)^3}$

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3. Use the change of base formula to write the following logarithms as logarithms in the indicated base.

a) $\log_7 12$; write it in base 10. **Solution:** $\log_7 12 = \dfrac{\log 12}{\log 7}$

b) $\log_9 127$; write it in base e. **Solution:** $\log_9 127 = \dfrac{\mathrm{Ln}\,127}{\mathrm{Ln}\,9}$

c) $\log_{25} 9$; write it in base 5 and simplify. **Solution:** $\log_5 9 = \dfrac{\log_5 9}{\log_5 25} = \dfrac{\log_5 9}{2}$

d) $\log_{\frac{1}{2}} 8$; write it in base 2 and simplify. **Solution:** $\log_{\frac{1}{2}} 8 = \dfrac{\log_2 8}{\log_2 \frac{1}{2}} = \dfrac{3}{-1} = -3$

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4. If $\log_2 b = \pi$, use the change of base formula to find $\log_b 8 = \dfrac{\log_2 8}{\log_2 b} = \dfrac{3}{\pi}$.

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5. Solve the following equations.

a) $4^x = 32$.

b) $27^x = 81$.

c) $2^{2x-1} = 32$

d) $5^{2-x} = \dfrac{1}{125}$

e) $7^{\frac{x-2}{6}} = \sqrt{7}$.

f) $8^{1-x} = 4^{x+2}$

Solutions:

a) $4^x = 32$. Write both sides as powers of 2: $(2^2)^x = 2^5$, so using the rules of exponents we have $2^{2x} = 2^5$, so $2x = 5$, so $x = \frac{5}{2}$.

b) $27^x = 81$. Write both sides as powers of 3: $(3^3)^x = 3^4$, so using the rules of exponents we have $3^{3x} = 3^4$, so $3x = 4$, so $x = \frac{4}{3}$.

c) $2^{2x-1} = 8$. Write both sides as powers of 2 and proceed as in the previous exercises. An alternative way follows: take $\log_2$ of both sides, $\log_2 2^{2x-1} = \log_2 8$, and simplify: $2x - 1 = 3$. Therefore $2x = 4$, so $x = 2$.

d) $5^{2-x} = \dfrac{1}{125}$. Proceed as in exercises **a)** and **b)** or as in **c**: Take $\log_5$ of both sides and simplify: $\log_5 5^{2-x} = \log_5 \dfrac{1}{125}$, so $2 - x = -3$. Then solve the equation: $2 = x - 3$, so $5 = x$.

e) $7^{\frac{x-2}{6}} = \sqrt{7}$. As before, take $\log_7$ of both sides and simplify: $\log_7 7^{\frac{x-2}{6}} = \log_7 \sqrt{7}$ so $\dfrac{x-2}{6} = \frac{1}{2}$. Solve this equation: multiply both sides by 6 to get $x - 2 = 3$, so $x = 5$.

f) $8^{1-x} = 4^{x+2}$. Take $\log_2$ of both sides (why base 2? Because 2 divides the bases of the exponentials in both sides): $\log_2 8^{1-x} = \log_2 4^{x+2}$. Use the properties of logarithms and simplify: $(1 - x)\log_2 8 = (x + 2)\log_2 4$. Now, $\log_2 8 = 3$ and $\log_2 4 = 2$, so we get $3(1 - x) = 2(x + 2)$, so $3 - 3x = 2x + 4$ so $-5x = 1$, so $x = -\frac{1}{5}$.

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6. Solve each exponential equation. Express each solution using natural logarithms (i.e. in base e) or logarithms in base 10. Then use a calculator to find a decimal approximation, correct to two decimal places.

a) $5e^x = 7$
b) $4e^{7x} = 10,273$
c) $3^{\frac{x}{7}} = 0.2$
d) $7^{2x-1} = 3^{x+2}$

Solutions:

a) $5e^x = 7$. Take Ln of both sides: $\text{Ln}(5e^x) = \text{Ln}\,7$. Use the properties of logarithms to expand: $\text{Ln}\,5 + \text{Ln}\,e^x = \text{Ln}\,7$. Now, $\text{Ln}\,e^x = x$, so we get $\text{Ln}\,5 + x = \text{Ln}\,7$. Therefore $x = \text{Ln}\,7 - \text{Ln}\,5 \approx 0.34$.

b) $4e^{7x} = 10,273$. Take Ln of both sides: $\text{Ln}(4e^{7x}) = \text{Ln}\,10,273$. Use the properties of logarithms to expand: $\text{Ln}\,4 + \text{Ln}\,e^{7x} = \text{Ln}\,10,273$. Now, $\text{Ln}\,e^{7x} = 7x$, so we get $\text{Ln}\,4 + 7x = \text{Ln}\,10,273$. Therefore $7x = \text{Ln}\,10,273 - \text{Ln}\,4$, so $x = \dfrac{\text{Ln}\,10,273 - \text{Ln}\,4}{7} \approx 1.12$.

c) $3^{\frac{x}{7}} = 0.2$. Take Ln of both sides: $\text{Ln}(3^{\frac{x}{7}}) = \text{Ln}\,0.2$. Use the properties of logarithms to expand: $\frac{x}{7}\,\text{Ln}\,3 = \text{Ln}\,0.2$. Divide both sides by Ln 3 to get $\frac{x}{7} = \frac{\text{Ln}\,0.2}{\text{Ln}\,3}$. Multiply both sides by 7 to get $x = \frac{7\,\text{Ln}\,0.2}{\text{Ln}\,3} \approx 10.25$.

d) $7^{2x-1} = 3^{x+2}$. Take Ln of both sides and use the properties of logarithms to get $(2x-1)\,\text{Ln}\,7 = (x+2)\,\text{Ln}\,3$. Distribute: $(2\,\text{Ln}\,7)x - \text{Ln}\,7 = x\,\text{Ln}\,3 + 2\,\text{Ln}\,3$. Subtract $x\,\text{Ln}\,3$ from both sides and add $\text{Ln}\,7$ to both sides: $(2\,\text{Ln}\,7)x - x\,\text{Ln}\,3 = \text{Ln}\,7 + 2\,\text{Ln}\,3$. Factor x out in the LHS: $x(2\,\text{Ln}\,7 - \text{Ln}\,3) = \text{Ln}\,7 + 2\,\text{Ln}\,3$ and finally divide both sides by $(2\,\text{Ln}\,7 - \text{Ln}\,3)$ to get $x = \dfrac{\text{Ln}\,7 + 2\,\text{Ln}\,3}{2\,\text{Ln}\,7 - \text{Ln}\,3} \approx 1.48$

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7. Solve the following logarithmic equations.

a) $\log_5 x = 3$
b) $\log_4(x - 7) = 3$
c) $2\log_5 x = 4$
d) $\log_5 x - 2 = \log_5 3$
e) $\log_2 \sqrt{x + 4} = 1$
f) $\log_2(x - 1) + \log_2(x + 1) = 3$
g) $\log(x + 7) - \log 3 = \log(7x - 1)$
h) $\log(x + 3) + \log(x - 2) = \log 14$

Solutions:

a) $\log_5 x = 3$. Write it in exponential form: $x = 5^3 = 125$.
Check the solution: $\log_5 5^3 = 3$: YES.

b) $\log_4(x - 7) = 3$. Write it in exponential form: $x - 7 = 4^3 = 64$. Then add 7 to both sides to get $x = 71$.

Check the solution: $\log_4(71-3) = \log_4 64 = 3$: YES.

c) $\log_5 x - 2 = \log_5 3$. Move the 2 to the right hand side and the $\log_5 3$ to the left hand side to get $\log_5 x - \log_5 3 = 2$. Then use the properties of logarithms to condense the LHS: $\log_5(\frac{x}{3}) = 2$. Now write it in exponential form: $\frac{x}{3} = 5^2 = 25$. Finally multiply by 3 to get $x = 75$.
Check the solution: $\log_5 75 - 2 = \log_5(25 \cdot 3) - 2 = \log_5 25 + \log_5 3 - 2 = 2 + \log_5 3 - 2 = \log_5 3$: YES.

d) $2\log_5 x = 4$. Divide both sides by 2: $\log_5 x = 2$, then write it in exponential form to get $x = 5^2 = 25$.
Check the solution: $2\log_5 25 = 2 \cdot 2 = 4$: YES.

e) $\log_2 \sqrt{x+4} = 1$. Write it in exponential form: $\sqrt{x+4} = 2$. Then square both sides to get rid of the $\sqrt{}$: $x+4 = 4$. Therefore, $x = 0$.
Check the solution: $\log_2 \sqrt{0+4} = \log_2 \sqrt{4} = \log_2 3 = 1$: YES.

f) $\log_2(x-1) + \log_2(x+1) = 3$. Use the properties of logarithms to condense the LHS: $\log_2(x-1)(x+1) = 3$. Then write it in exponential form: $(x-1)(x+1) = 2^3 = 8$. Now solve this equation. First expand the LHS to get $x^2 - 1 = 8$, so $x^2 = 9$, so $x = -3$ or $x = 3$.
Check the solution: Check the solution $x = -3$: $\log_2(-3-1) + \log_2(-3+1) = \log_2(-4) + \log_2(-2)$ which is NOT DEFINED, so $x = -3$ is NOT a solution. Now check $x = 3$: $\log_2(3-1) + \log_2(3+1) = \log_2(2) + \log_2(4) = 1 + 2 = 3$: YES. Therefore the only solution is $x = 3$.

g) $\log(x+7) - \log 3 = \log(7x-1)$. Use the properties of logarithms to condense the LHS: $\log \frac{x+7}{3} = \log(7x-1)$. Remove the log's in both sides: $\frac{x+7}{3} = 7x - 1$. Then solve this equation: multiply both sides by 3 to get $x + 7 = 21x - 3$, so $-20x = -10$, so $x = \frac{1}{2}$.
Check the solution: The LHS is $\log\left(\frac{1}{2} + 7\right) - \log 3 = \log \frac{15}{2} - \log 3 = \log \frac{15}{6} = \log \frac{5}{3}$. The RHS is $\log\left(7 \cdot \frac{1}{2} - 1\right) = \log\left(\frac{7}{2} - 1\right) = \log \frac{5}{2}$: YES.

h) $\log(x+3) + \log(x-2) = \log 14$. Use the properties of logarithms to condense the LHS: $\log(x+3)(x-2) = \log 14$. Remove the log's in both sides to get $(x+3)(x-2) = 14$. Now solve this equation: expand the LHS to get $x^2 + x - 6 = 14$, so $x^2 + x - 20 = 0$. Factor the LHS: $(x+5)(x-4) = 0$ so the possible solutions are $x = -5$ and $x = 4$.
Check the solution: First check $x = -5$: $\log(-5+3) + \log(-5-2) = \log(-2) + \log(-7)$ which is undefined, so $x = -5$ is NOT a solution. Now check $x = 4$: $\log(4+3) + \log(4-2) = \log(7) + \log(2) = \log(7 \cdot 2) = \log 14$: YES. Therefore the only solution is $x = 4$.