MATH 30 - Precalculus. Homework 5. Due Th. 03/14/2024. Professor Luis Fernández

SOLUTION

DO NOT write your answers here, except the graphs. Do it in other sheets and **show all your work**.
STAPLE this sheet to your other sheets.

1. For the following rational functions, first find

1. The end behaviour and the horizontal asymptotes, if any.
2. The vertical asymptotes.
3. The x-intercepts and their multiplicity.
4. The y-intercept.

Then **sketch the graph of the function** in the graph paper provided (or in your own).

a) $f(x) = \dfrac{x+1}{x-1}$

b) $f(x) = \dfrac{3x^2}{x^2-9}$

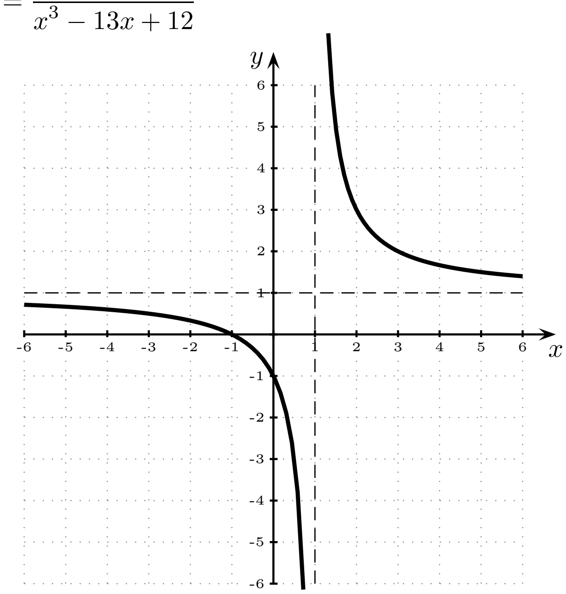
c) $f(x) = \dfrac{x-4}{x^2-x-6}$

$f(x) = \dfrac{2x+5}{x^3-13x+12}$

Solutions:

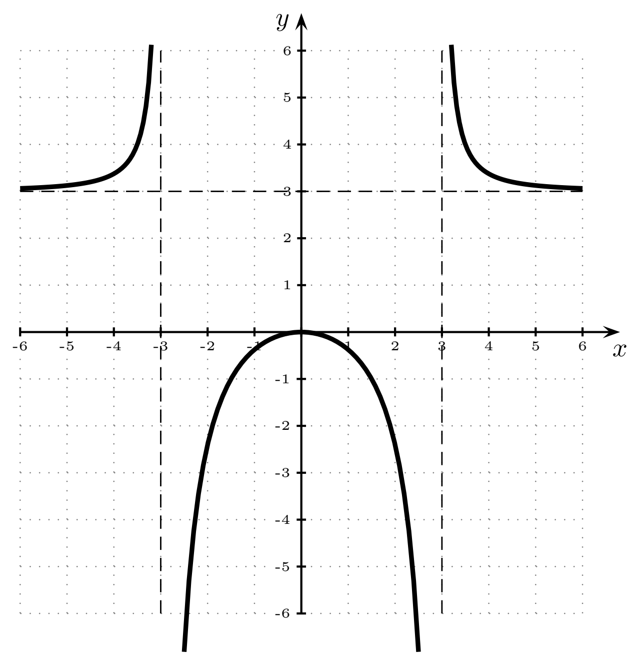
a)

1. End behaviour: As $x \to \pm\infty$, $f(x) \approx \frac{x}{x} = 1$, so it has a horizontal asymptote at $y = 1$.
2. Vertical asymptotes: the denominator is $x - 1$; it is 0 only when $x = 1$, so f has a vertical asymptote at $x = 1$.
3. x-intercepts: the numerator is $x+1$; it is 0 only when $x = -1$, so the only x-intercept is $x = -1$ with multiplicity 1.
4. y-intercept: $f(0) = \frac{0+1}{0-1} = -1$.

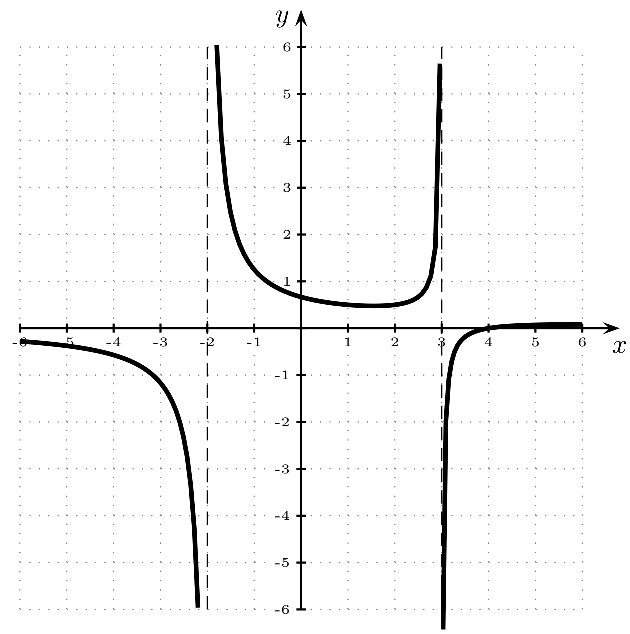


b)

1. End behaviour: As $x \to \pm\infty$, $f(x) \approx \frac{3x^2}{x^2} = 3$, so it has a horizontal asymptote at $y = 3$.
2. Vertical asymptotes: the denominator is $x^2 - 9$; it is 0 only when $x = 3$ or $x = -3$, so f has vertical asymptotes at $x = 3$ and at $x = -3$.
3. x-intercepts: the numerator is x^2; it is 0 only when $x = 0$, so the only x-intercept is $x = 0$ with multiplicity 2.
4. y-intercept: $f(0) = 0$.

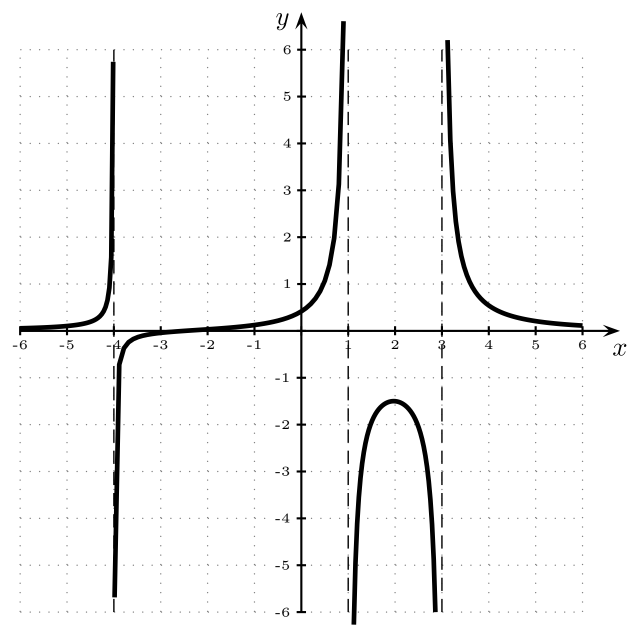


c)



1. <u>End behaviour</u>: As $x \to \pm\infty$, $f(x) \approx \frac{x}{x^2} = \frac{1}{x}$, which has a horizontal asymptote at $y = 0$. Therefore f has also a horizontal asymptote at $y = 0$.
2. <u>Vertical asymptotes</u>: the denominator is $x^2 - x - 6 = (x-3)(x+2)$, so f has vertical asymptotes at $x = 3$ and $x = -2$.
3. <u>x-intercepts</u>: the numerator is $x - 4$, so the only x-intercept is $x = 4$ with multiplicity 1.
4. <u>y-intercept</u>: $f(0) = \frac{0-4}{0^2 - 0 - 6} = \frac{-4}{-6} = \frac{2}{3}$.

d)



1. <u>End behaviour</u>: As $x \to \pm\infty$, $f(x) \approx \frac{2x}{x^3} = \frac{2}{x^2}$, which has a horizontal asymptote at $y = 0$. Therefore f has also a horizontal asymptote at $y = 0$.
2. <u>Vertical asymptotes</u>: the denominator is $x^3 - 13x + 12 = (x-1)(x-3)(x+4)$, so f has vertical asymptotes at $x = 3$ and $x = -4$ and $x = 1$.
3. <u>x-intercepts</u>: the numerator is $2x+5$, so the only x-intercept is $x = -\frac{5}{2} = -2.5$ with multiplicity 1.
4. <u>y-intercept</u>: $f(0) = \frac{5}{12}$.

---

2. Use a calculator to approximate the following numbers to 4 decimal places.

a) $2^{3.4} = 10.5560$ **b)** $e^{1.5} = 4.4817$ **c)** $6^{-\frac{1}{3}} = 1.8171$ **d)** $\sqrt{3}^{\sqrt{2}} = 2.1746$

e) $\log 12 = 1.0792$ **f)** $\log \sqrt{5} = 0.3495$ **g)** $\ln \frac{1}{5} = 1.6094$ **h)** $\ln 469993 = 13.0605$

---

3. Find without using a calculator.

a) $\log_2 8 = 3$ **b)** $\log_3 \frac{1}{3} = -1$ **c)** $\log_6 \sqrt{6} = \frac{1}{2}$ **d)** $\log_{102} 102^4 = 4$

e) $\log_8 2 = \frac{1}{3}$ **f)** $\log_{27} \frac{1}{3} = -\frac{1}{3}$ **g)** $\log_5 1 = 0$ **h)** $\log_3(\log_8 2) = \log_3(\frac{1}{3}) = -1$

---

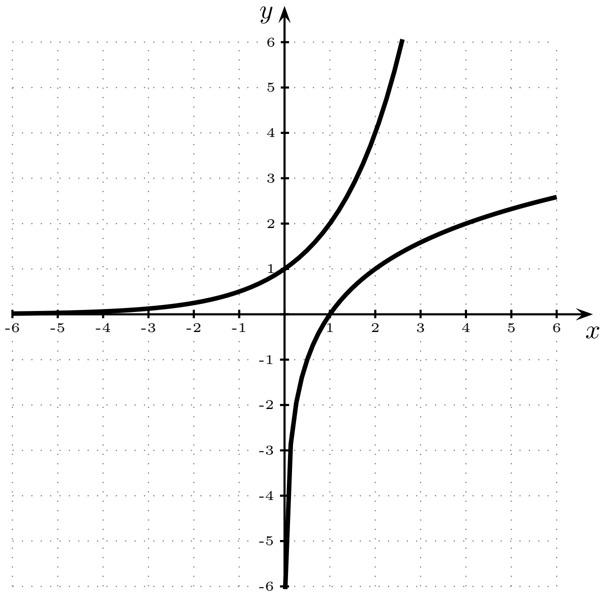
4. Simplify each expression. Here a is a positive number.

a) $\log_a a^4 = 4$ **b)** $\log_a \frac{1}{a^7} = -7$ **c)** $\log_a a^{\frac{1}{5}} = \frac{1}{5}$ **d)** $\log_a \sqrt[3]{a} = \frac{1}{3}$

e) $2^{\log_2 7} = 7$ **f)** $a^{\log_a \frac{1}{5}} = \frac{1}{5}$ **g)** $10^{\log \sqrt{4}} = \sqrt{4}$ **h)** $e^{\ln 3x^2} = 3x^2$

5. Graph the following functions in the axes provided (both in the same axes).

a) $f(x) = 2^x$ and $g(x) = \log_2 x$.



b) $f(x) = 2^{x-3} - 5$ and $g(x) = \log_2(x+5) + 3$.