MATH 30 - Precalculus. Homework 4. Due We. 03/06/2024. Professor Luis Fernández

SOLUTION

DO NOT write your answers here. Do it in other sheets and **show all your work**.
STAPLE this sheet to your other sheets.

1. Use synthetic division and the remainder theorem to find the indicated function value.

a) $f(x) = x^3 - 4x^2 + x + 2$; find $f(3)$.

b) $f(x) = -2x^4 - x^2 + x - 2$; find $f(-1)$.

c) $f(x) = x^5 - 4x^2 + 1$; find $f(2)$.

d) $f(x) = -x^4 - 5x^3 - x^2 + 3x + 2$; find $f\left(\frac{1}{2}\right)$.

Solution:
Synthetic divisions skipped. For example, you can use http://www.mathcelebrity.com/syndiv.php to check it.

a) Answer: $f(3) = -4$.

b) Answer: $f(-1) = -6$.

c) Answer: $f(2) = 17$.

d) Answer: $f(1/2) = \frac{41}{16}$.

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2. Solve the following polynomial equations. (We did several examples in class.)

a) $x^3 - 4x^2 - 7x + 10 = 0$

b) $3x^3 - 8x^2 - 8x + 8 = 0$

c) $x^4 + 3x^3 - 20x^2 + 24x - 8 = 0$

d) $x^4 - x^3 + 2x^2 - 4x - 8 = 0$

Solution:

a) The possible rational solutions of $x^3 - 4x^2 - 7x + 10 = 0$ are ± 1, ± 2, ± 5, ± 10. Now we do synthetic division to test them. Check that 1 is not a root. However, -1 is:

$$\begin{array}{r|rrrr}
 & 1 & -4 & -7 & 10 \\
1 & & 1 & -3 & -10 \\
\hline
 & 1 & -3 & -10 & 0 \\
-2 & & -2 & 10 & \\
\hline
 & 1 & -5 & 0 & \\
5 & & 5 & & \\
\hline
 & 1 & 0 & &
\end{array}$$

Therefore the solutions are 1, -2 and 5.

b) I only write the solutions (proceed as in the previous exercise, or as in exercise 2). They are $\frac{2}{3}$, $1 + \sqrt{5}$, $1 - \sqrt{5}$.

c) I only write the solutions (proceed as in the previous exercise, or as in exercise 2). They are: 1, 2, $-3 - \sqrt{13}$, $-3 + \sqrt{13}$.

d) I only write the solutions (proceed as in the previous exercise, or as in exercise 2). They are -1, 2, $2i$, $-2i$.

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3. Use the results of the previous exercise to factor the following polynomials completely.
[NOTE: you DO NOT need to do *any* calculation, only use the *factor theorem*.]

a) $x^3 - 4x^2 - 7x + 10$

b) $3x^3 - 8x^2 - 8x + 8$

c) $x^4 + 3x^3 - 20x^2 + 24x - 8$

d) $x^4 - x^3 + 2x^2 - 4x - 8$

Solution:

a) From the previous exercise, the zeros of $x^3 - 4x^2 - 7x + 10$ are 1, -2 and 5.
Therefore, $x^3 - 4x^2 - 7x + 10 = (x - 1)(x + 2)(x - 5)$

b) From the previous exercise, the zeros of $3x^3 - 8x^2 - 8x + 8$ are $\frac{2}{3}$, $1 + \sqrt{5}$, $1 - \sqrt{5}$.
Therefore, $3x^3 - 8x^2 - 8x + 8 = (x - \frac{2}{3})(x - (1 + \sqrt{5}))(x - (1 - \sqrt{5}))$.

c) From the previous exercise, the zeros of $x^4 + 3x^3 - 20x^2 + 24x - 8$ are 1, 2, $-3 - \sqrt{13}$, $-3 + \sqrt{13}$.
Therefore, $x^4 + 3x^3 - 20x^2 + 24x - 8 = (x - 1)(x - 2)(x - (-3 - \sqrt{13}))(x - (-3 + \sqrt{13}))$

d) From the previous exercise, the zeros of $x^4 - x^3 + 2x^2 - 4x - 8$ are -1, 2, $2i$, $-2i$.
Therefore, $x^4 - x^3 + 2x^2 - 4x - 8 = (x + 1)(x - 2)(x + 2i)(x - 2i)$.

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4. Solve the equation $(x - 1)^2(x - 2)(x - 3)(x + 4) = 0$.
[NOTE: you DO NOT need to do any calculation for this one; use the *factor theorem* to find the solution by just looking at the equation.]
Solution: 1 (with multiplicity two), 2, 3 and -4.

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5. Find the possible rational zeros of the following polynomials.

a) $4x^3 + 5x^2 - 3x + 6$

b) $6x^4 + 3x^2 + 4x - 15$

Solution:

a) $\pm\{1, 2, 3, 6, \frac{1}{2}, \frac{1}{4}, \frac{3}{2}, \frac{3}{4}\}$

b) $\pm\{1, 3, 5, 15, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}, \frac{3}{2}, \frac{5}{2}, \frac{5}{3}, \frac{5}{6}, \frac{15}{2}\}$